Short Course on Experimental Dynamic Substructuring

Module #7a: Estimating Fixed-Interface Modes from Measurements on a Flexible Fixture

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Short Course Notes For:
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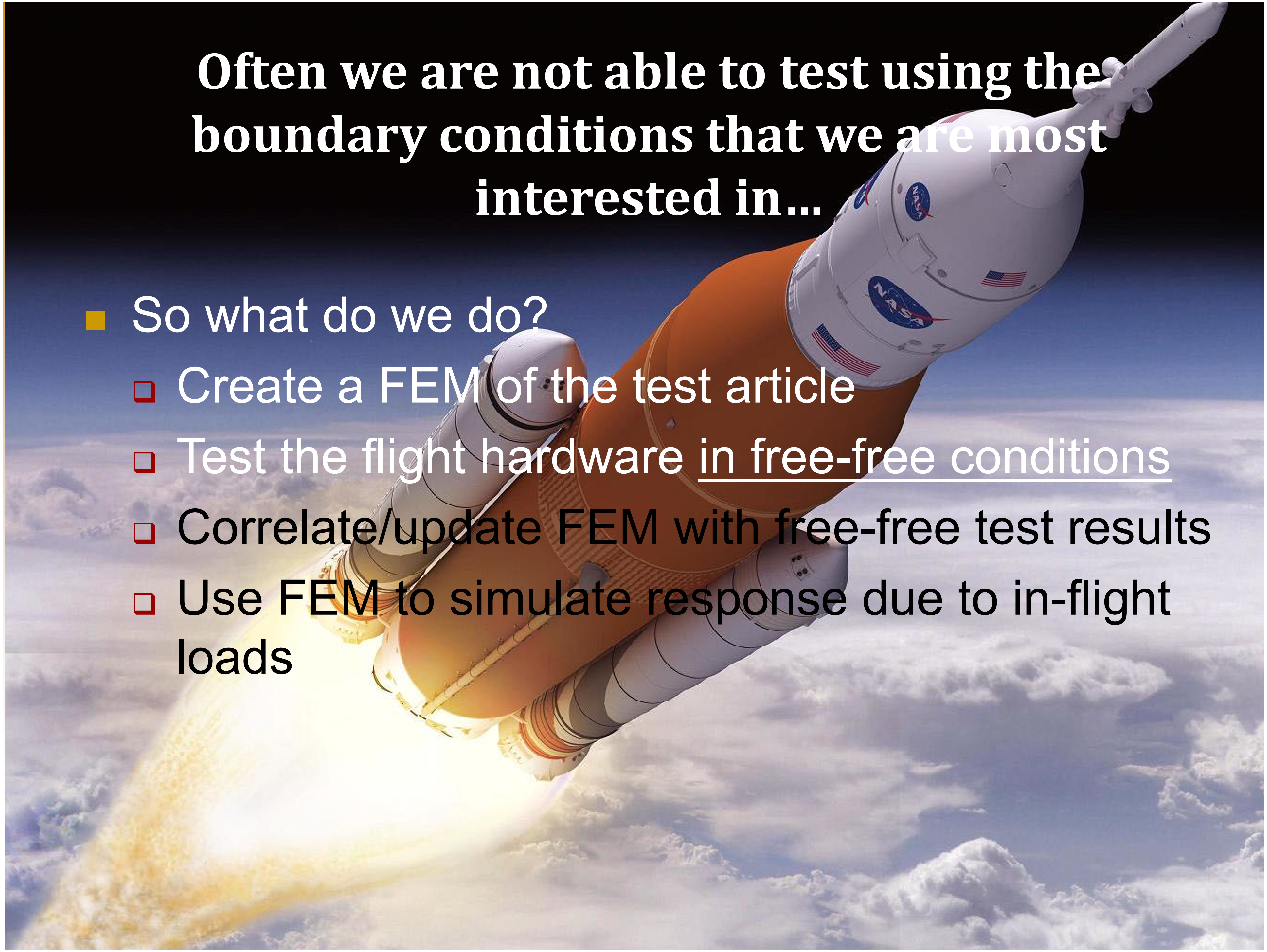


Often we are not able to test using the boundary conditions that we are most interested in…

- So what do we do?
  - Create a FEM of the test article
  - Test the flight hardware <u>in free-free conditions</u>
  - Correlate/update FEM with free-free test results
  - Use FEM to simulate response due to in-flight loads

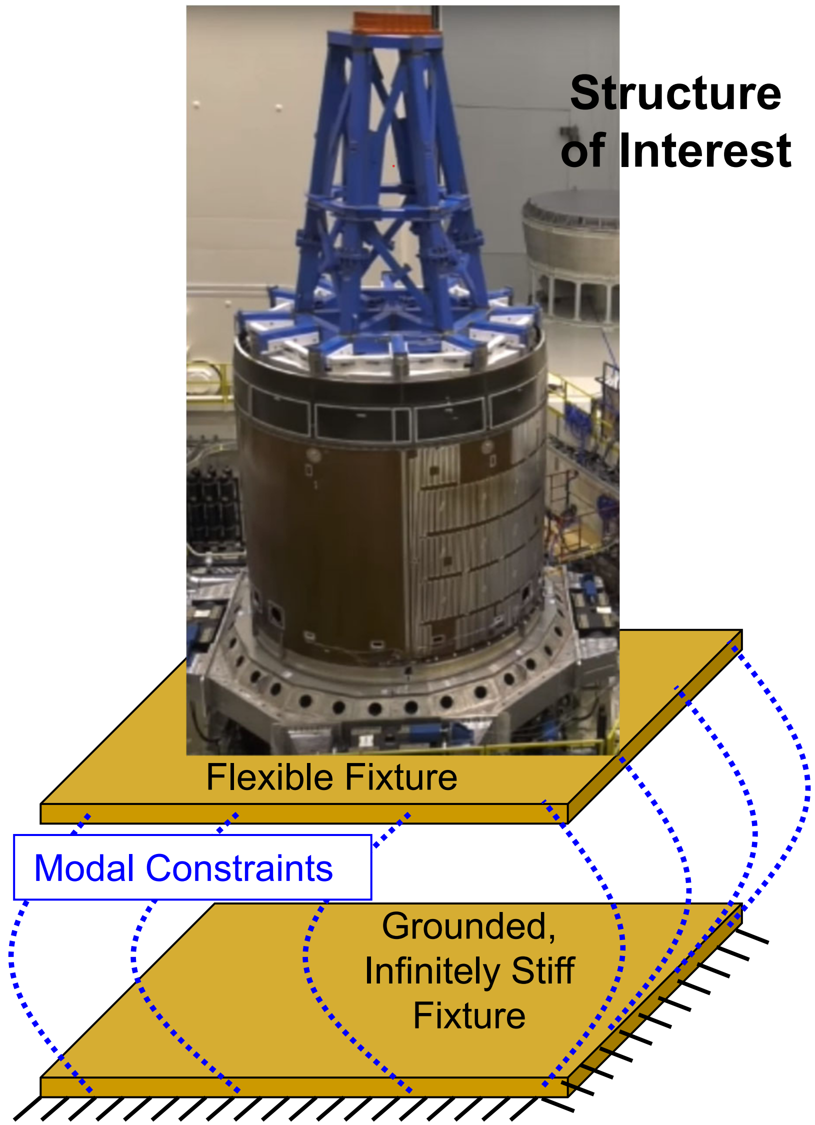
A Free-Free test of the SLS is not feasible. Can we estimate Fixed-Interface modes from a modal test of the SLS+ML

Integrated Modal Test (IMT) will be performed on SLS+ML

Other Applications

- Test campaigns often include:
  - Low level vibration test to obtain modal parameters.
  - Testing on vibration tables at higher amplitudes to test durability.
- It would be preferable to combine these two to reduce cost, test time and risk of damaging the test article.
  - Then one could validate the model for the system using fixed-base modes.

https://www.youtube.com/watch?v=SX3P-foK_mw



Outline

- Modal Substructuring Theory
  - Modal constraints vs. connection point constraints
- Experiment: Simple plate-beam system
  - Conventional connection point method
  - Modal Constraints
  - Fixture Design
- Experiment: Wind Turbine Blade
- Experiment: Slip Table Modal Test (Sandia)
- Conclusions

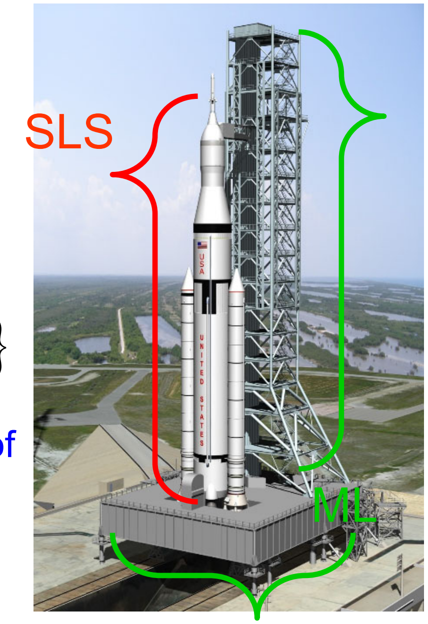
Modal Substructuring Theory

- Equations of Motion for (Fixture+Substructure) assembly found experimentally:

$$\left[I \right]\{\ddot{\mathbf{q}}\}+\left[\backslash 2\zeta_r\omega_r \backslash \right]\{\dot{\mathbf{q}}\}+\left[\backslash \omega_r^{2} \backslash \right]\{\mathbf{q}\}=\left[\boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}} \quad \boldsymbol{\varphi}_{\mathrm{s}}^{\mathrm{T}} \right]\{\mathbf{F}\}$$

Fixture (circling $\boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}}$); Substructure (circling $\boldsymbol{\varphi}_{\mathrm{s}}^{\mathrm{T}}$)

i.e. modes of SLS+ML



- Modal Parameters of Fixture alone:

i.e. modes of ML alone $\quad \boldsymbol{\varphi}^{\mathrm{fixt}}{}_{\mathrm{f}} \qquad \mathbf{q}^{\mathrm{fixt}}$

- Fixed-base modes of the substructure are those for which the motion of the fixture is zero.

$$\mathbf{y}^{\mathrm{fixt}}=0 \quad \Rightarrow \quad \mathbf{q}^{\mathrm{fixt}}=0$$

- Use the following constraint equations to make the Fixture motion zero on the (Fixture+Substructure):

$$\mathbf{y}^{\mathrm{fixt}} \approx \boldsymbol{\varphi}^{\mathrm{fixt}}{}_{f}\,\mathbf{q}^{\mathrm{fixt}} \qquad \left[\left(\boldsymbol{\varphi}^{\mathrm{fixt}}{}_{\mathrm{f}} \right)^{+} \boldsymbol{\varphi}_{\mathrm{f}} \right] \mathbf{q}=0$$

$$\mathbf{B}\mathbf{q}=0$$

$$\mathbf{B}\mathbf{L}\mathbf{q}_u=0$$

$$\mathbf{L}=null(\mathbf{B})$$

$$\hat{\mathbf{K}}=\mathbf{L}^{\mathrm{T}}\left[\backslash \omega_r^{2} \backslash \right]\mathbf{L}$$

etc…

$$\hat{\mathbf{M}}\ddot{\mathbf{q}}_u+\hat{\mathbf{C}}\dot{\mathbf{q}}_u+\hat{\mathbf{K}}\mathbf{q}_u=\mathbf{L}^{\mathrm{T}}\left[\boldsymbol{\varphi}_{\mathrm{f}}^{\mathrm{T}} \quad \boldsymbol{\varphi}_{\mathrm{s}}^{\mathrm{T}} \right]\{\mathbf{F}\}$$

Modal Substructuring Theory

Physical Constraints

$$\mathbf{y}_{\mathrm{CPT}} = 0$$

Modal Constraints

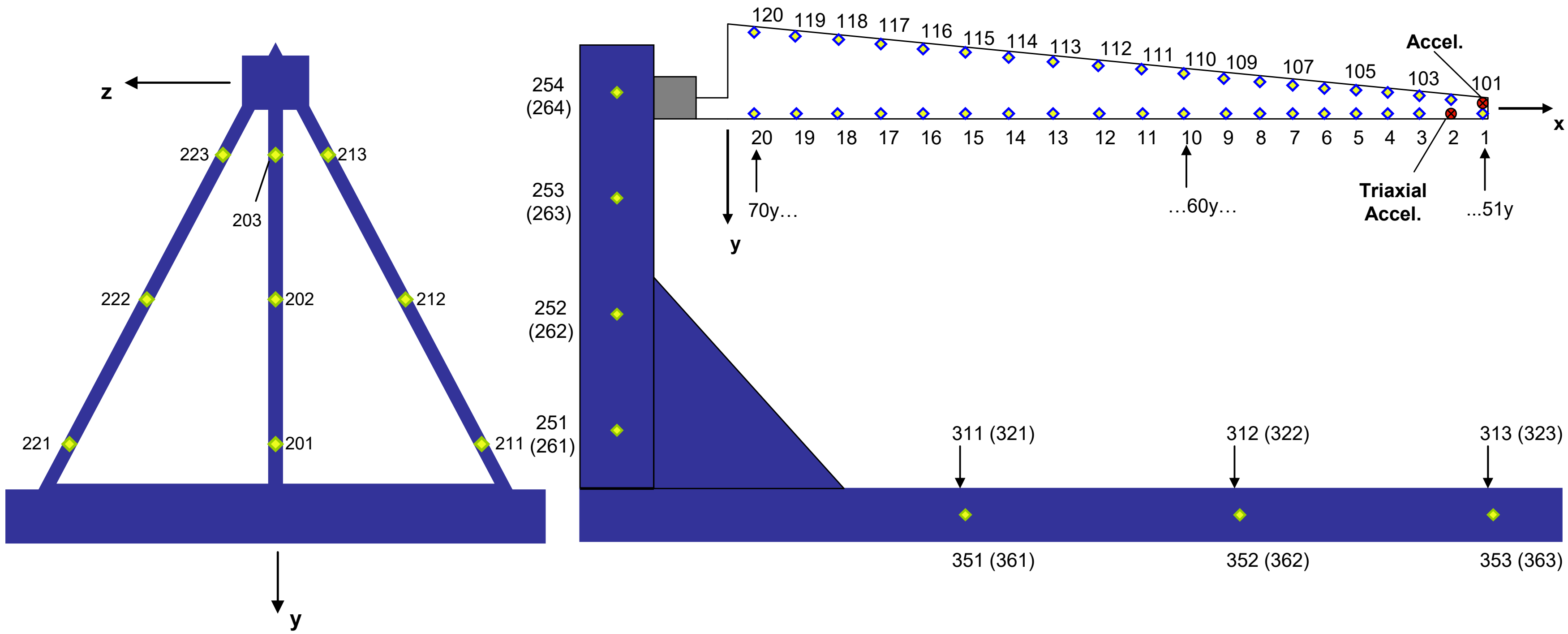
$$\mathbf{q}^{\mathrm{fixt}} = 0$$

$$\mathbf{\Phi}_{\mathrm{CPT}}\left[\left(\mathbf{\Phi}^{\mathrm{fixt}}{}_{\mathrm{f}}\right)^{+}\mathbf{\Phi}_{\mathrm{f}}\right]\mathbf{q} = 0$$

$$\left[\left(\mathbf{\Phi}^{\mathrm{fixt}}{}_{\mathrm{f}}\right)^{+}\mathbf{\Phi}_{\mathrm{f}}\right]\mathbf{q} = 0$$

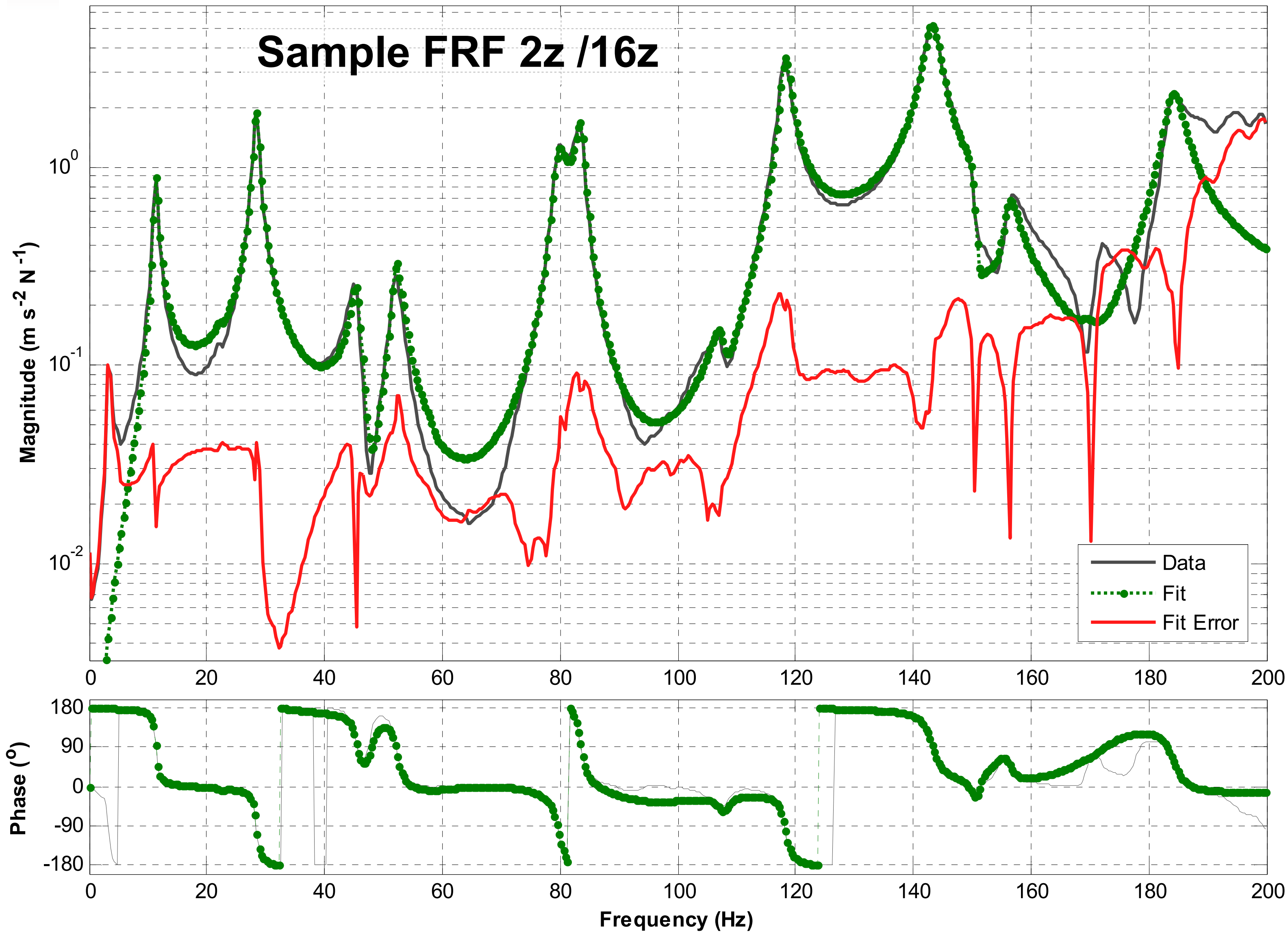
- The matrix $\left(\mathbf{\Phi}^{\mathrm{fixt}}{}_{\mathrm{f}}\right)^{+}$ is an orthogonal projector. Any of the measured motions in the space of the projector (i.e. spanned by the modes of the fixture) will be set to zero.

Modal Test on Wind Turbine Blade



- Test performed in a stiff frame that was used for static fatigue testing.
- Desire to correlate modes with a fixed-base model for the blade.

Curve fit performed on all measurements



Constraints enforced on modal model identified from the measurements.

Modal Constraints

$$\mathbf{q}^{\text{fixt}} = 0$$

$$\left[\left(\boldsymbol{\Phi}^{\text{fixt}}{}_{\text{f}}\right)^{+} \boldsymbol{\Phi}_{\text{f}}\right]\mathbf{q} = 0$$

- Modes of the test fixture were not known, so instead an SVD was applied to the matrix of measured modes on the fixture

All Measured Modes

Partitioned to fixture DOF only

$$(\boldsymbol{\Phi}_{\text{f}}) = \mathbf{USV}^{\mathbf{T}}$$

$$[\mathbf{U}^{+}\boldsymbol{\Phi}_{\text{f}}]\mathbf{q} = 0$$

$$\mathbf{Bq} = 0$$

$$\mathbf{BLq}_u = 0$$

$$\mathbf{L} = null(\mathbf{B})$$

$$\hat{\mathbf{K}} = \mathbf{L}^{\text{T}}\left[\setminus \omega_r^{2} \setminus\right]\mathbf{L}$$

$$\hat{\mathbf{M}}\ddot{\mathbf{q}}_u + \hat{\mathbf{C}}\dot{\mathbf{q}}_u + \hat{\mathbf{K}}\mathbf{q}_u = \mathbf{L}^{\text{T}}\left[\boldsymbol{\varphi}_{\text{f}}{}^{\text{T}} \quad \boldsymbol{\varphi}_{\text{s}}{}^{\text{T}}\right]\{\mathbf{F}\}$$

Modes After Applying SVD Constraints

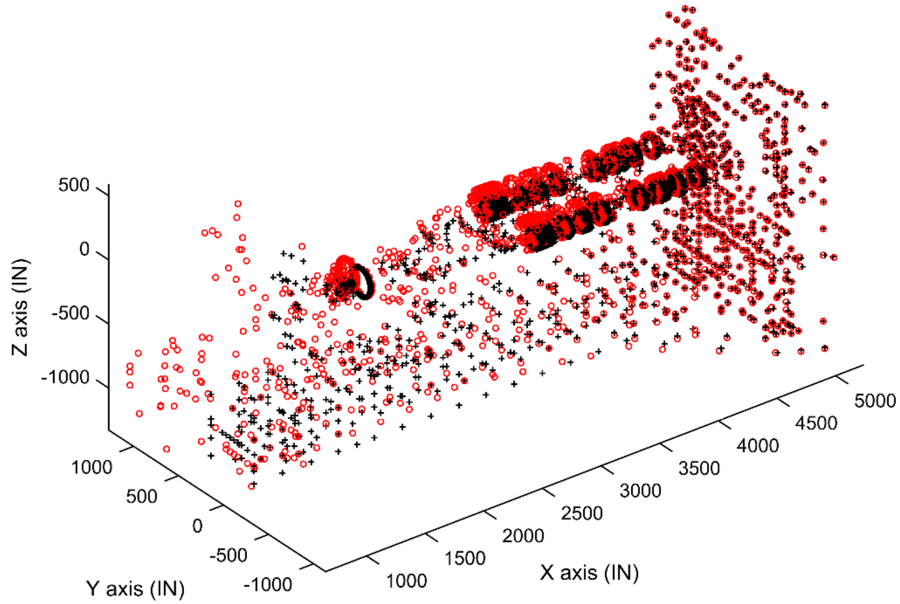
Mode Num.	Mode Shape Description	Natural Freq. (Hz)	2 SVD Constraints		3 SVD Constraints	
			f_n CMS	% Diff	f_n CMS	% Diff
1	FW B1	3.36	3.83	12.1%	3.84	12.4%
2	EW B1	5.24	5.27	0.5%	5.28	0.7%
3	FW B2	11.40	11.44	0.4%	11.64	2.1%
4	EW B2	22.42	22.52	0.4%	22.77	1.6%
5	FW B3	28.44	28.85	1.4%	29.54	3.7%
6	FW B4, Fixture+	45.50	48.92	7.0%	50.26	9.5%
7	FW B4, Fixture–	52.26	-	-	-	-
8	EW+FW	53.37	-	-	-	-
9	EW B3	58.29	56.52	-3.1%	56.96	-2.3%
10	1st Torsion	80.01	79.96	-0.1%	79.97	0.0%
11	FW B5	83.54	81.84	-2.1%	83.90	0.4%
12	EW B4	107.37	106.85	-0.5%	107.01	-0.3%
13	FW B6	118.25	115.77	-2.1%	119.75	1.2%
14	2nd Torsion	143.47	143.45	0.0%	143.54	0.0%
15	FW B7, Tors.	150.29	150.12	-0.1%	154.12	2.5%
16	FW B7, Tors.	156.21	154.18	-1.3%	-	-
17	EW B5 +FW	169.61	168.30	-0.8%	159.09	-6.6%
18	FW B7, EW B8, Torsion	184.11	183.02	-0.6%	182.97	-0.6%

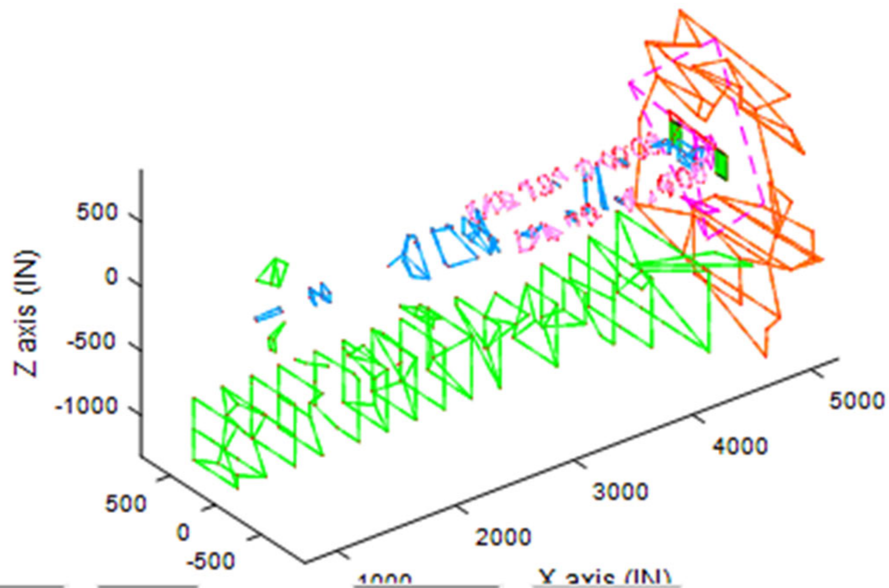
- Natural frequencies of wind turbine blade in frame.
- Modes correspond to:
  - Edgewise bending (EW)
  - Flapwise bending (FW)
  - Torsion.

Application to SLS Simulation Models

- Dry run for implementation as part of the Integrated Modal Test (IMT)

Simulation Model used for the Dry Lab

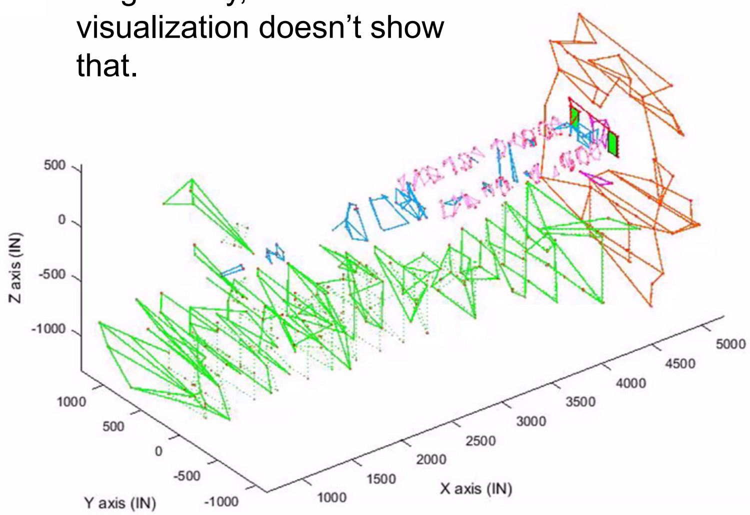


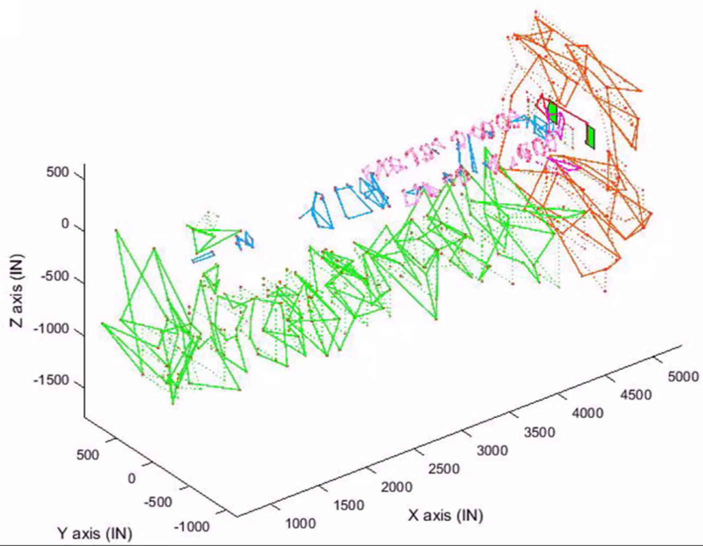


- **Test**: (**SLS+ML**)
  - Model of SLS+ML provided with 3926 DOF.
- **Truth Model: (SLS-Fixed)**
  - DOF on ML fixed to obtain a 2720 DOF model of the SLS with the **interface fixed**.
- **Test**: Measurements at 625 or 625+24 DOF
  - 374 DOF on SLS
  - 251 DOF on ML
  - 24 additional DOF at interface of SLS to ML
  - Use this to seek to estimate the modes of the SLS-Fixed

Application to SLS: Modes of ML with SLS Rigidized

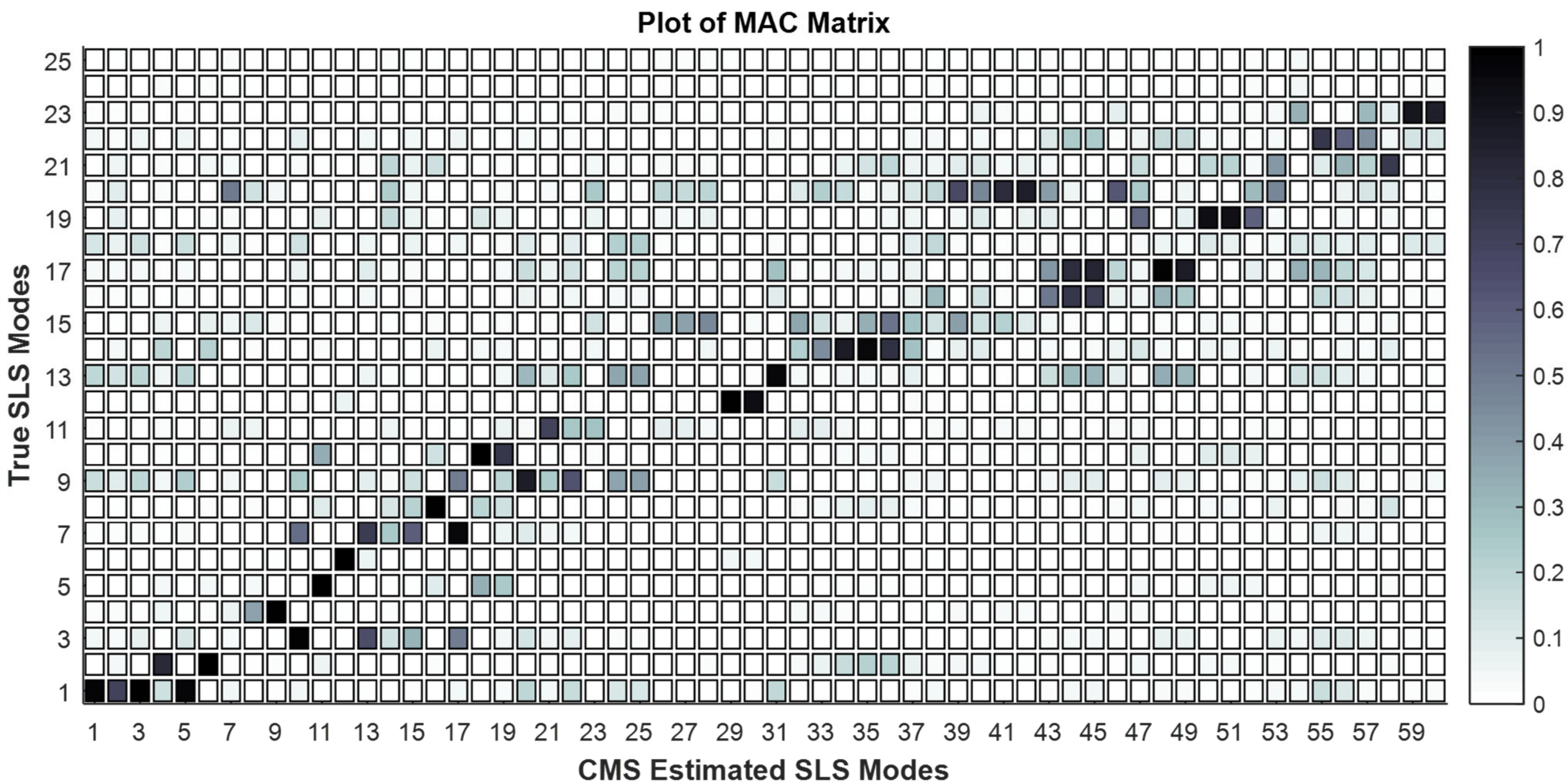
Note, the SLS is moving as a rigid body, but this visualization doesn't show that.





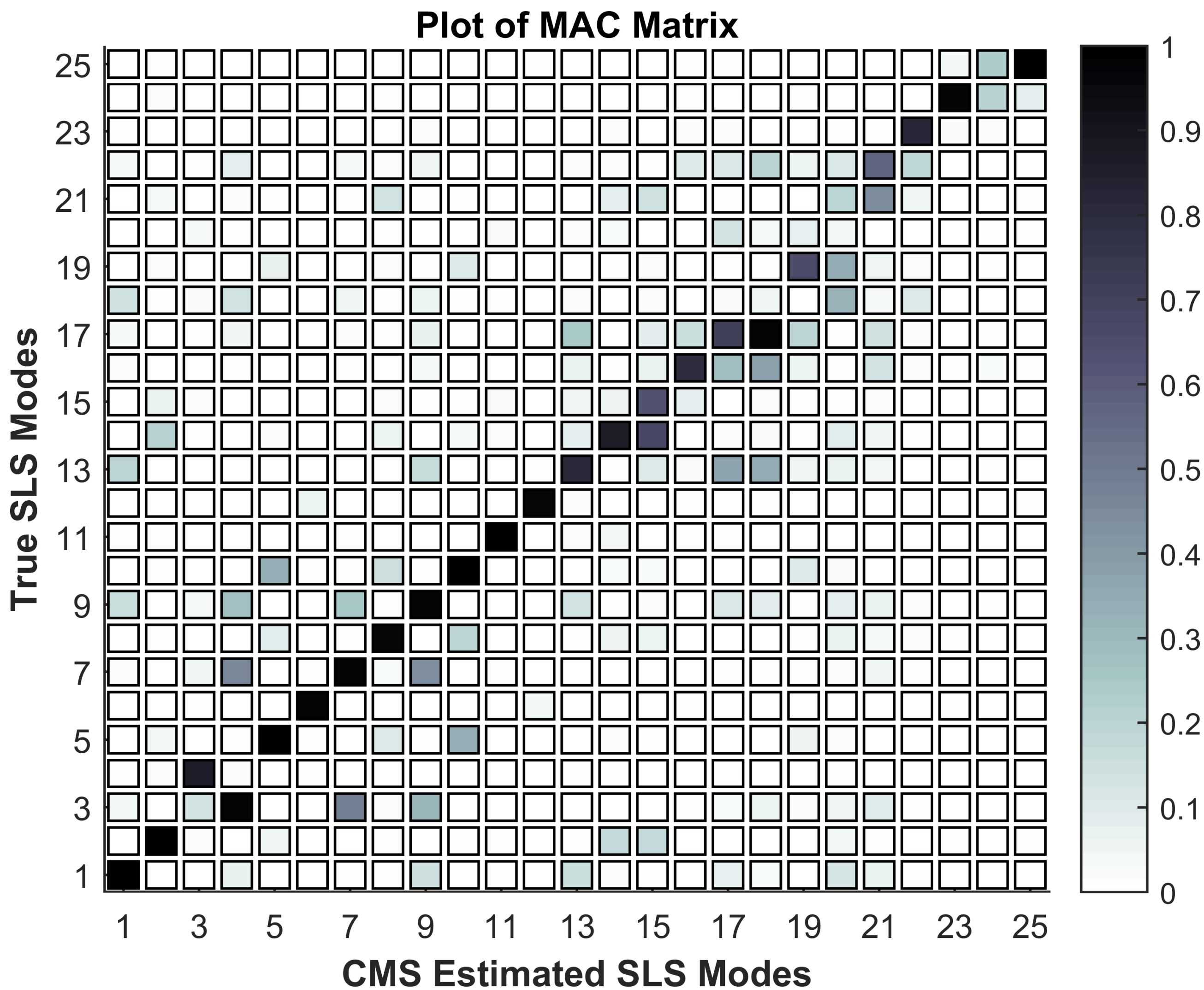
- All SLS DOF in the 3926 DOF model were rigidized and then the modes of the ML were estimated and used as a basis.
- Denoted ML-R Constraints.
  - 84 Constraint Modes Freq. of interest
- Note, the 24 interface DOF are shown in the visualization of the shapes but they were not used (presumed not measured).
- 3rd and 12th modes shown.

Result with Traditional Connection Point Constraints – MAC



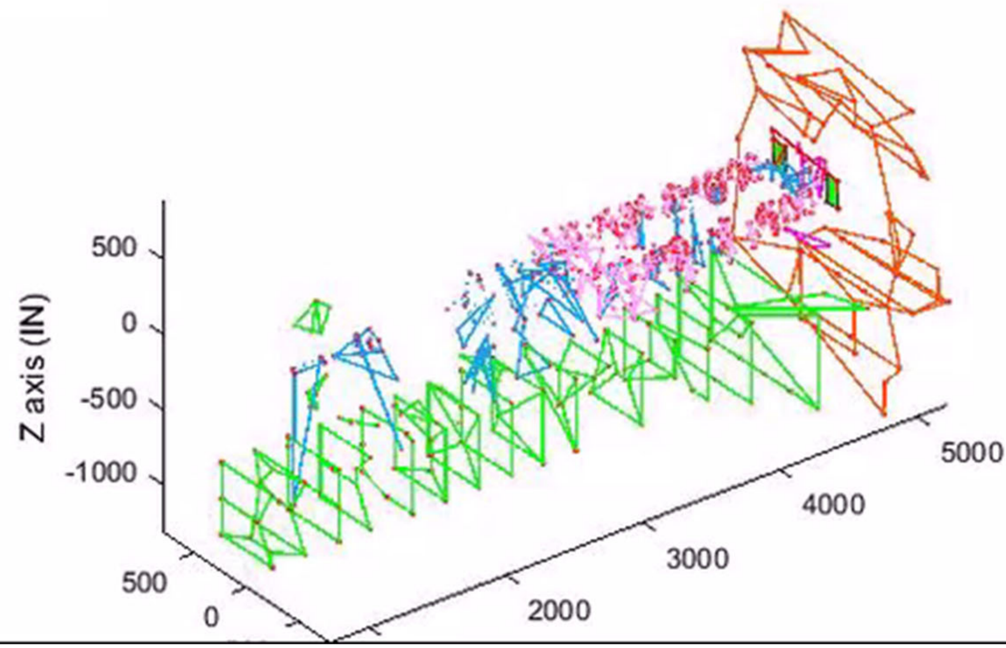
- The 6 constraints eliminate only 6 modes, so many spurious modes remain, most of which are dominated by motion of the ML.

Results – 84 ML-R Constraints - MAC

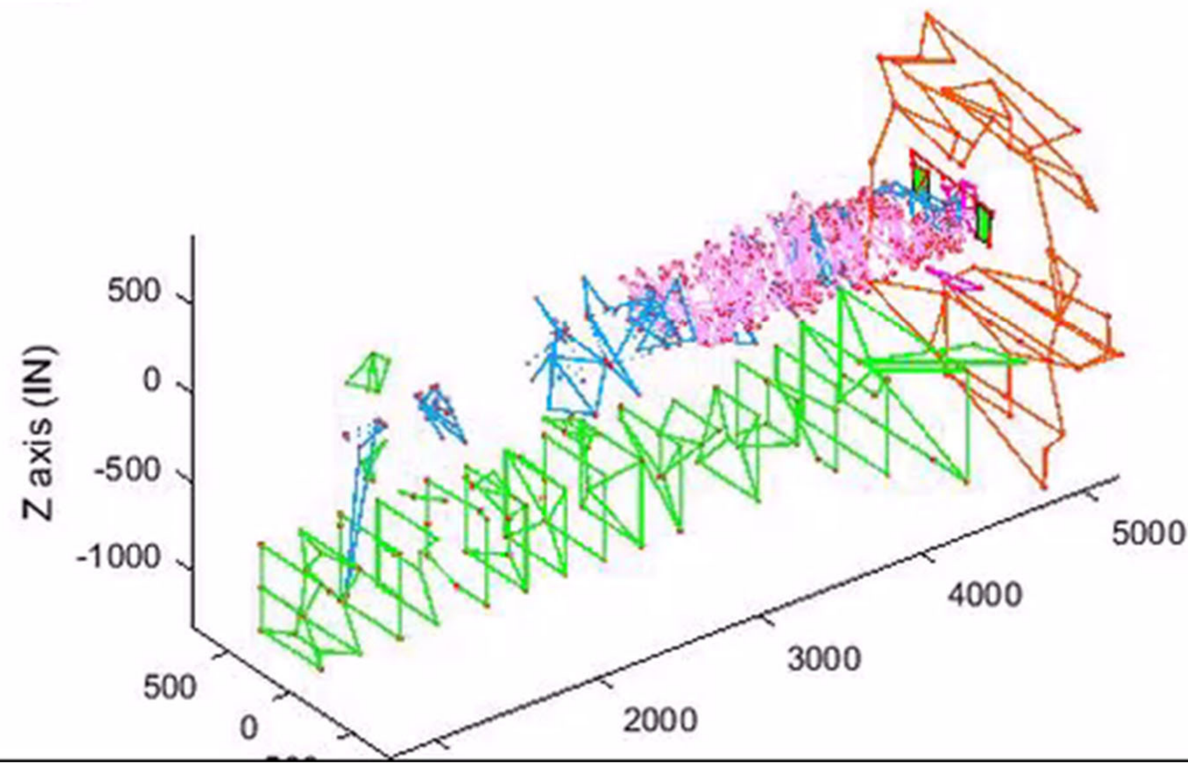


- Nearly one-to-one correspondence between modes!

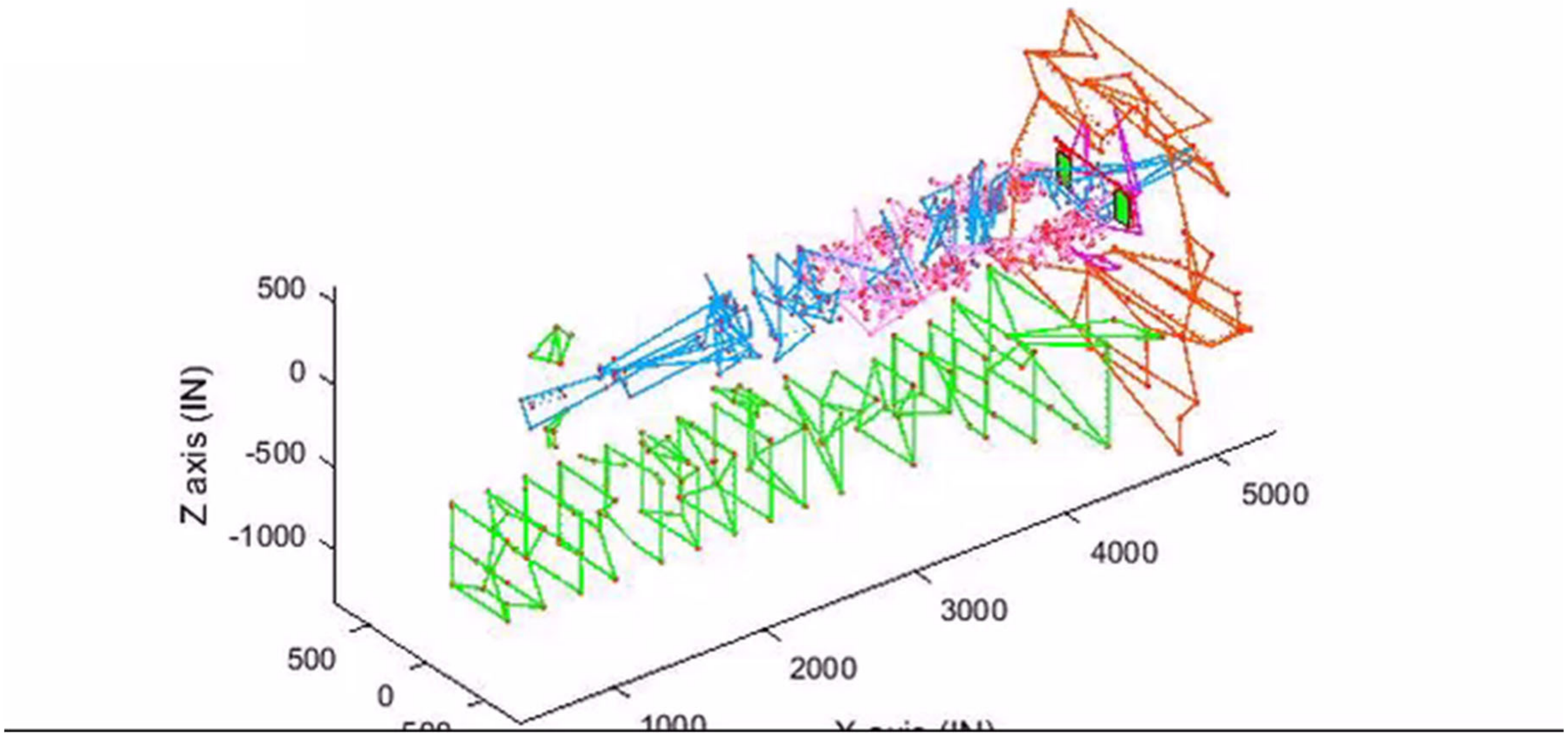
Mode 1 after applying 84 ML-R Constraints



Mode 3 after applying 84 ML-R Constraints

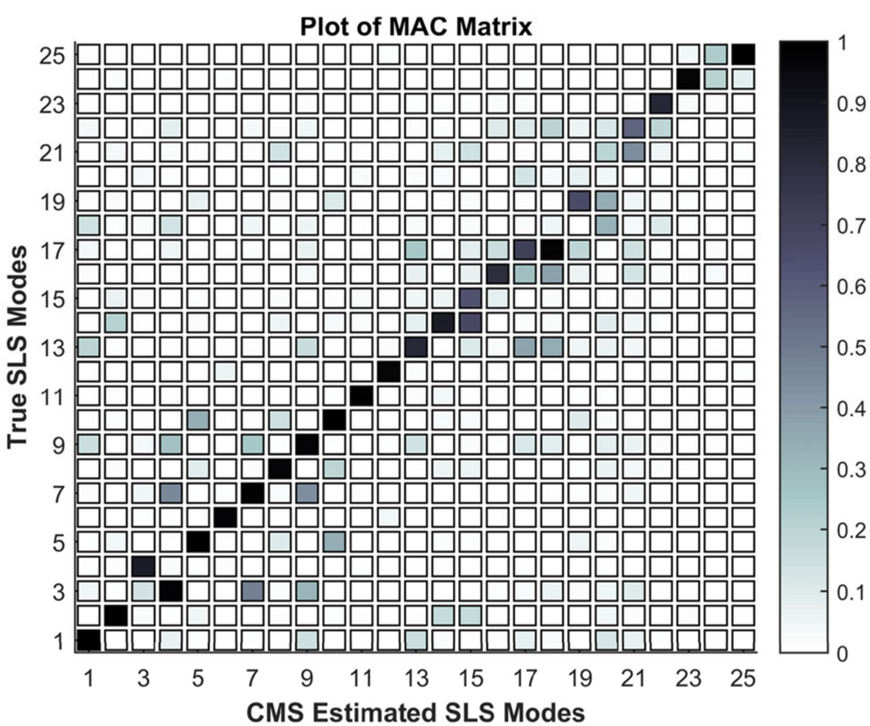


Mode 16 after applying 84 ML-R Constraints

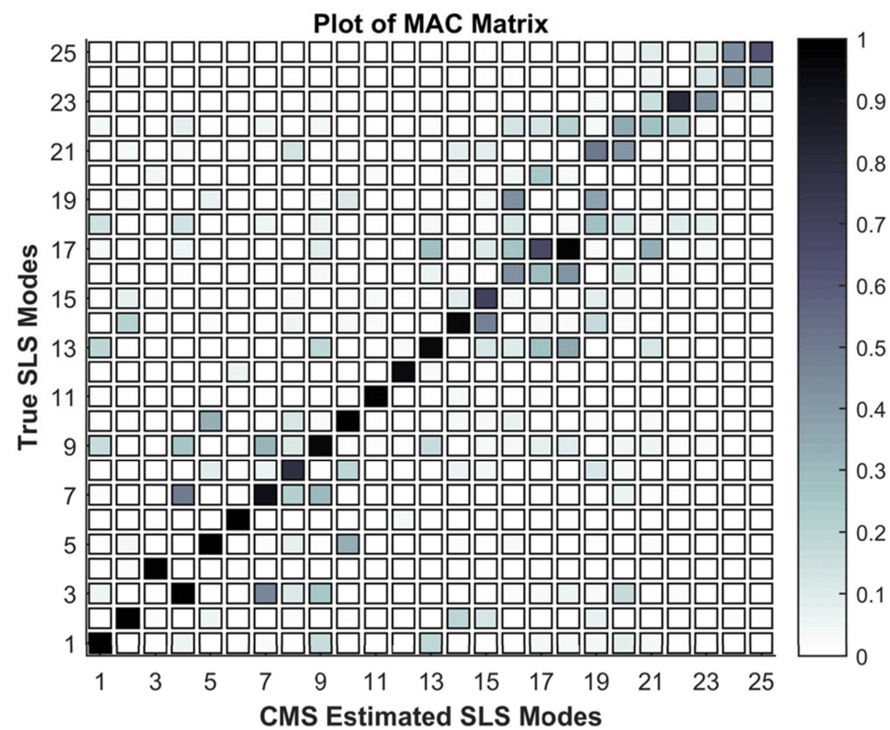


Comparison SLS Rigidized or Excluded

ML-R (i.e. SLS-Rigidized)



ML-F (i.e. w/out SLS)



Quantitative Comparison

SLS Mode #	Measured SLS+ML Freq. % Error	Measured SLS+ML MAC	84 Constraints: Modes of ML-R (i.e. SLS Rigidized) CMS Mode #	Freq. % Error	MAC	84 Constraints: Modes of ML-F (i.e. SLS Removed) CMS Mode #	Freq. % Error	MAC
1	**-12.4**	**1.00**	1	**-11.0**	**1.00**	1	**-2.6**	**1.00**
2	**-9.4**	**1.00**	2	**-5.1**	**1.00**	2	**-6.0**	**1.00**
3	**-8.7**	**0.92**	4	**-1.7**	**0.98**	4	**0.3**	**1.00**
4	**-6.8**	**0.95**	3	**-6.0**	**0.87**	3	**-5.9**	**0.99**
5	**-2.1**	**0.95**	5	**-0.5**	**1.00**	5	**-0.7**	**1.00**
6	**-3.4**	**1.00**	6	**-2.6**	**0.98**	6	**-3.3**	**0.99**
7	**1.4**	**0.99**	7	**-2.9**	**0.98**	7	**0.5**	**0.93**
8	**-1.6**	**0.94**	8	**-3.3**	**0.97**	8	**-3.2**	**0.79**
9	**-2.2**	**0.97**	9	**-2.3**	**0.98**	9	**0.6**	**0.96**
10	**-1.9**	**0.95**	10	**-0.1**	**1.00**	10	**-0.4**	**1.00**
11	**-2.1**	**0.97**	11	**-2.2**	**0.99**	11	**-1.9**	**0.99**
12	**-2.2**	**0.99**	12	**-2.8**	**0.97**	12	**-2.3**	**0.95**
13	**-6.3**	**0.98**	13	**-5.4**	**0.82**	13	**-5.8**	**0.95**
14	**-1.6**	**0.92**	14	**-1.8**	**0.87**	14	**-2.1**	**0.96**
15	**-6.5**	**0.76**	15	**-1.1**	**0.64**	15	**-1.1**	**0.70**
16	**-35.2**	**0.45**	16	**-1.2**	**0.80**	16	**6.3**	**0.43**
17	**-0.1**	**0.99**	18	**0.3**	**0.98**	18	**-0.3**	**0.98**
18	**0.8**	**0.39**	20	**-0.5**	**0.32**	20	**-1.2**	**0.27**
19	**-1.5**	**0.87**	19	**-4.8**	**0.67**	19	**-9.0**	**0.43**
20	**-6.9**	**0.70**	17	**-8.7**	**0.13**	17	**-7.7**	**0.24**

Free-Free modes of ML with a rigid SLS attached.

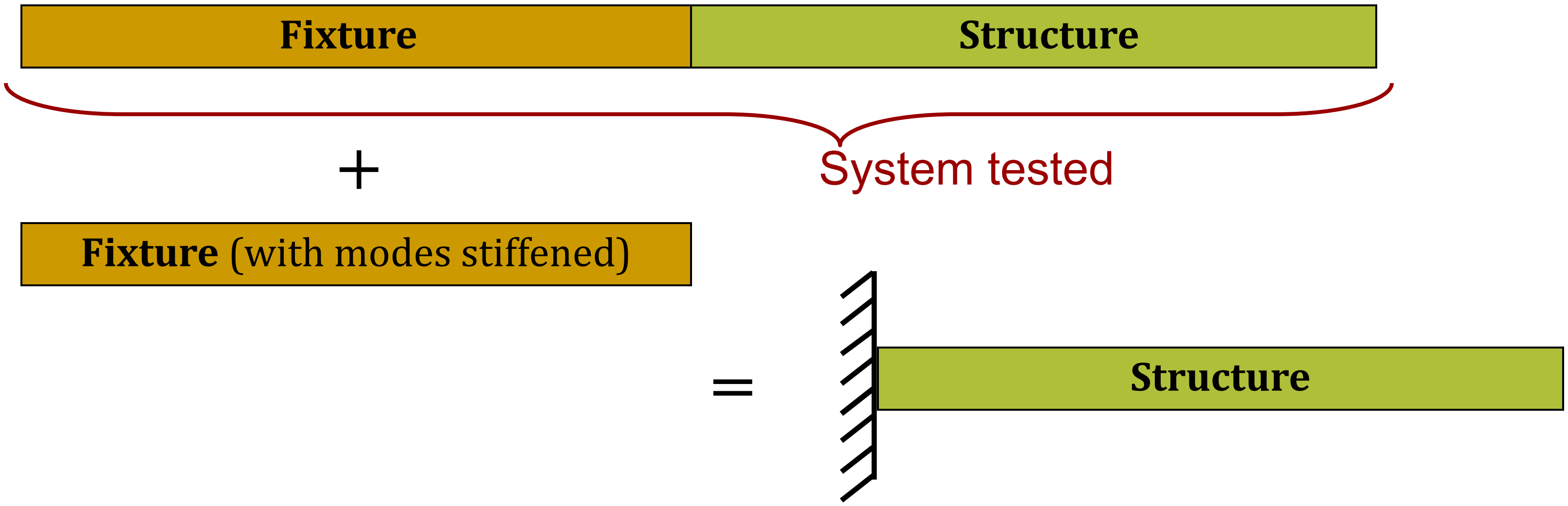
Free-Free modes of ML

Conclusions

- Objective: Estimate fixed-base modes of a structure from measurements on the structure and a fixture.
- Conventional Approach: Estimate the connection point displacement and rotation and use CMS to force them to zero.
  - Spurious modes were present in the CMS predictions.
  - There were significant errors in many of the natural frequencies.
- Modal Constraints: Estimate the modal motion of the fixture and force it to zero.
  - Several simulation and experimental case studies have shown that this works well.
    - Robust in the presence of measurement errors.
  - However, there can be relatively large errors in natural frequencies if the fixture is soft relative to the system of interest.
  - CMS converged smoothly as the number of modes increases, and the results were always qualitatively reasonable.

Further Examples

Simple Example - Beam



- First we considered a very simple example, a 12”x0.75”x1” steel beam.
  - A simulated test is performed with this beam coupled to another identical beam as shown.
  - We then use MCFS to estimate the fixed-base modes of the structure (beam on the right) from measurements on the test structure.

Note on constraints

- One way to constrain the fixture to ground:
  - $[\phi_f]^+\{y_f\} = [\phi_f]^+\{y_{f+s}\}$, where
    - $[\phi_f]$ = fixture mode shapes
    - $\{y_f\}$ = measurement points on fixture when it was tested in isolation
    - $\{y_{f+s}\}$ = measurement points on fixture when it was tested with the structure of interest attached
    - $()^+$ = pseudoinverse
  - This projects the motion of the (f+s) system onto each of the fixture modes and constrains the two. The fixture natural frequencies are then made very large to approximate a fixed base.
- Another way to do this is to constrain the projection of the fixture modes to zero (limit of an infinitely stiff fixture).
  - $[\phi_f]^+\{y_{f+s}\} = 0$

Natural Frequencies Before Coupling:

- Presuming that we can measure all of the modes of each system out to 8000 Hz, we obtain 5 and 7 modes with the following frequencies:

Mode	Fixture	F + S
1	0	0
2	0	0
3	1089.3	272.3
4	3002.8	750.7
5	5886.7	1471.7
6		2432.7
7		3634.1

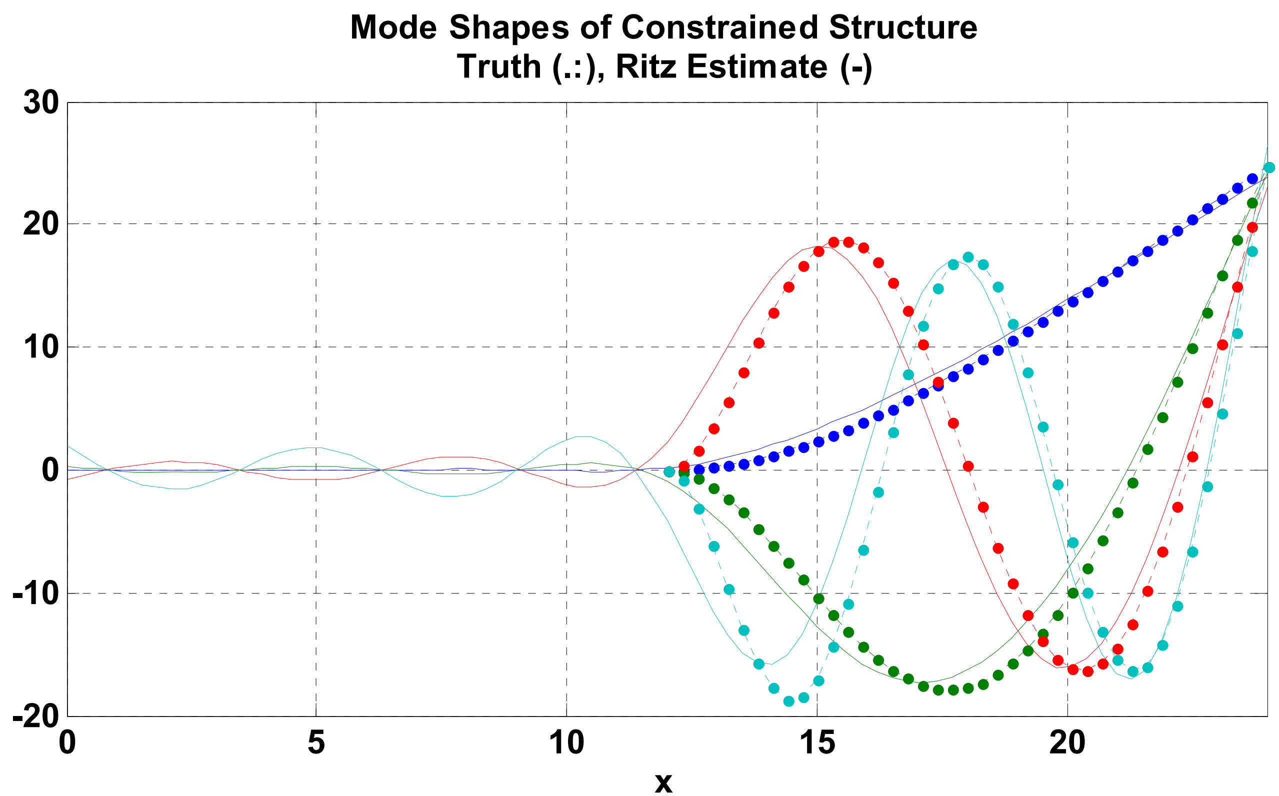
Modes after Coupling (5/7 Modes)

Modes of Fixed-Base Structure

Mode	Actual	CMS	% Error
1	171.2	145.6	-15.0
2	1072.8	924.7	-13.8
3	3004.0	2612.1	-13.0
4	5886.6	5140.8	-12.7

- Natural frequencies are in the ball park, but there are considerable errors.

- Mode shapes are similar to the true ones, but with only 5 constraints the motion of the fixture cannot be completely eliminated.



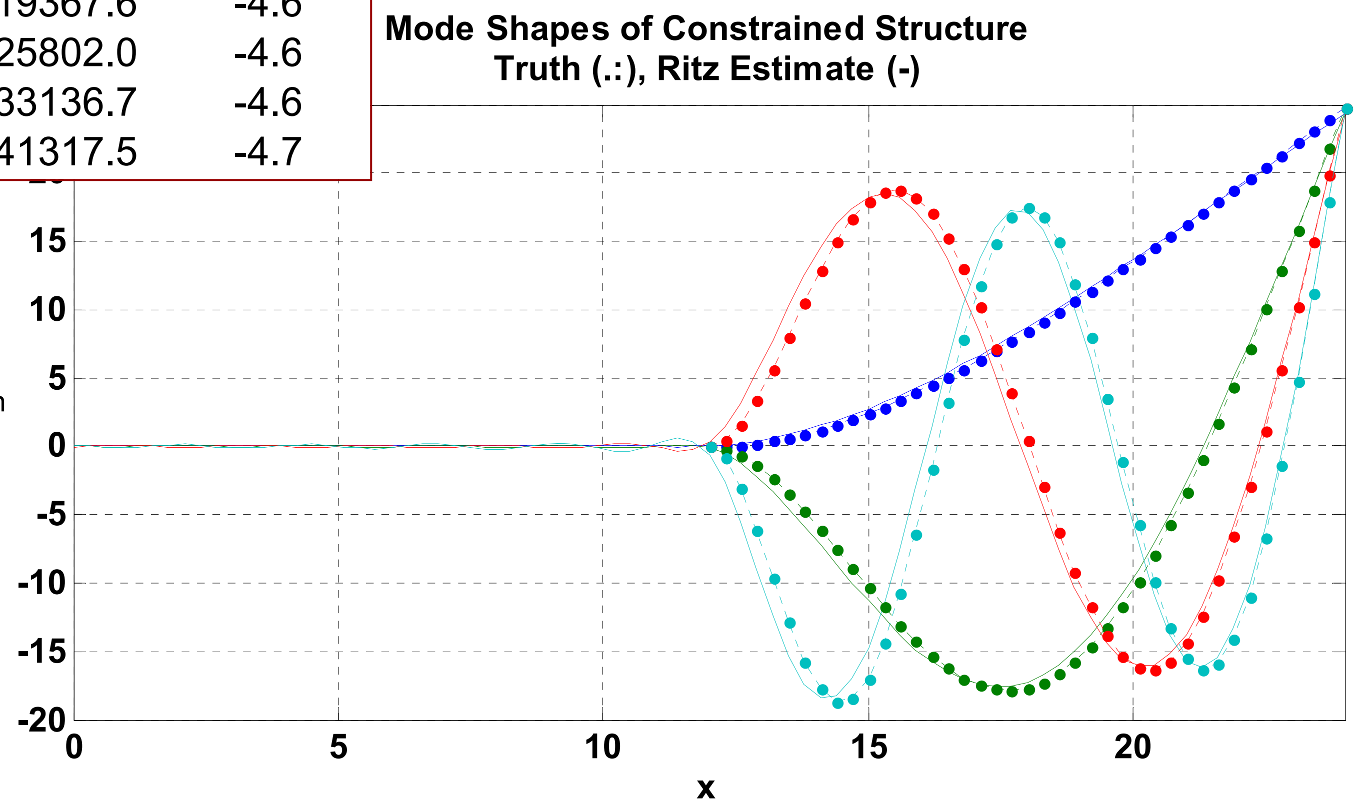
Modes after Coupling (11/21 Modes)

Modes of Fixed-Base Structure

Mode	Actual	CMS	% Error
1	171.2	161.4	-5.7
2	1072.8	1014.5	-5.4
3	3004.0	2846.9	-5.2
4	5886.6	5590.4	-5.0
5	9731.0	9258.1	-4.9
6	14536.6	13850.9	-4.7
7	20303.7	19367.6	-4.6
8	27032.5	25802.0	-4.6
9	34723.6	33136.7	-4.6
10	43377.7	41317.5	-4.7

- Everything is converging as the number of modes increases, but even with 11 and 21 modes (out to 45,000 Hz) there are 5% errors in the natural frequencies.

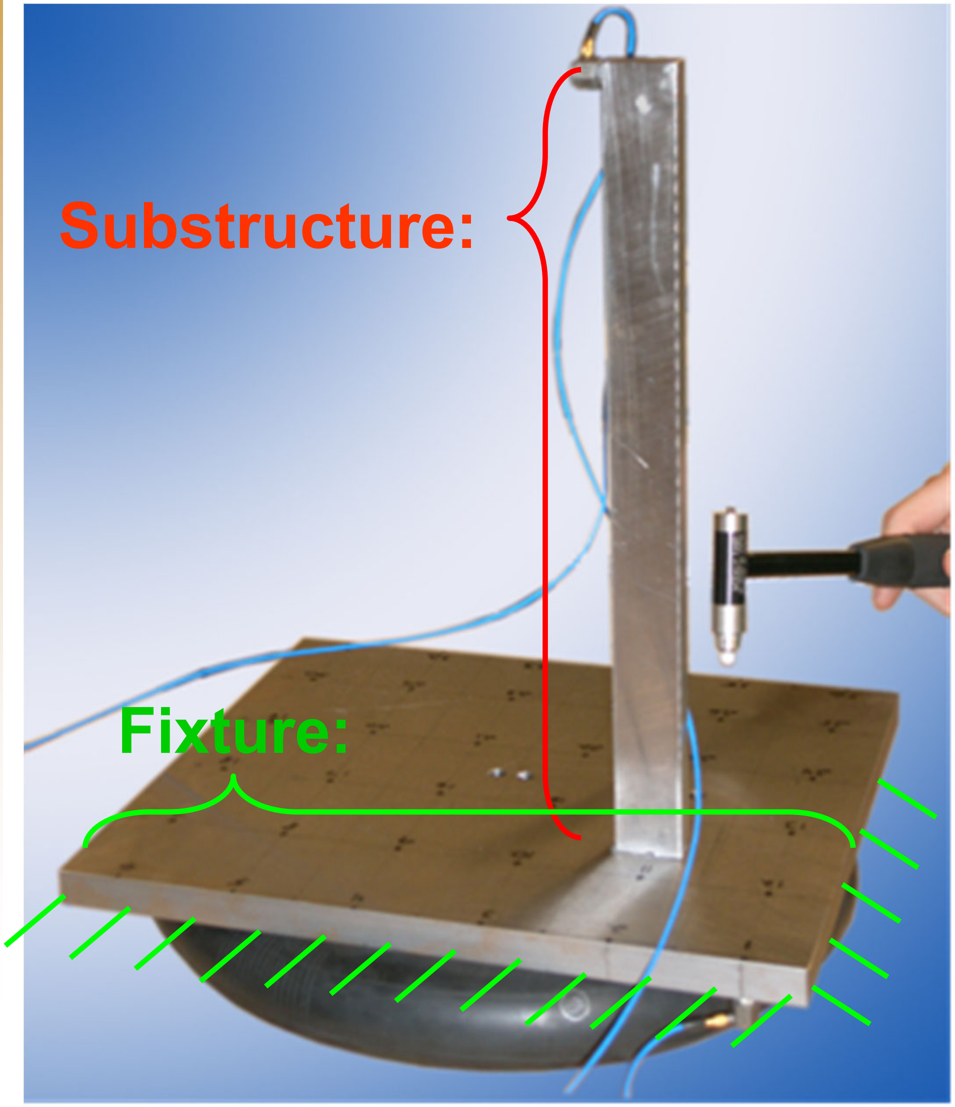
- This appears to be the usual issue of free-free modes converging slowly near an interface. Also, this is an extreme example where the fixed base is just as flexible as the structure of interest. The fixed base will usually be more rigid than the structure.



Appendix

- Application to Plate-Beam System

1st Test Case: Hardware / Experimental Setup



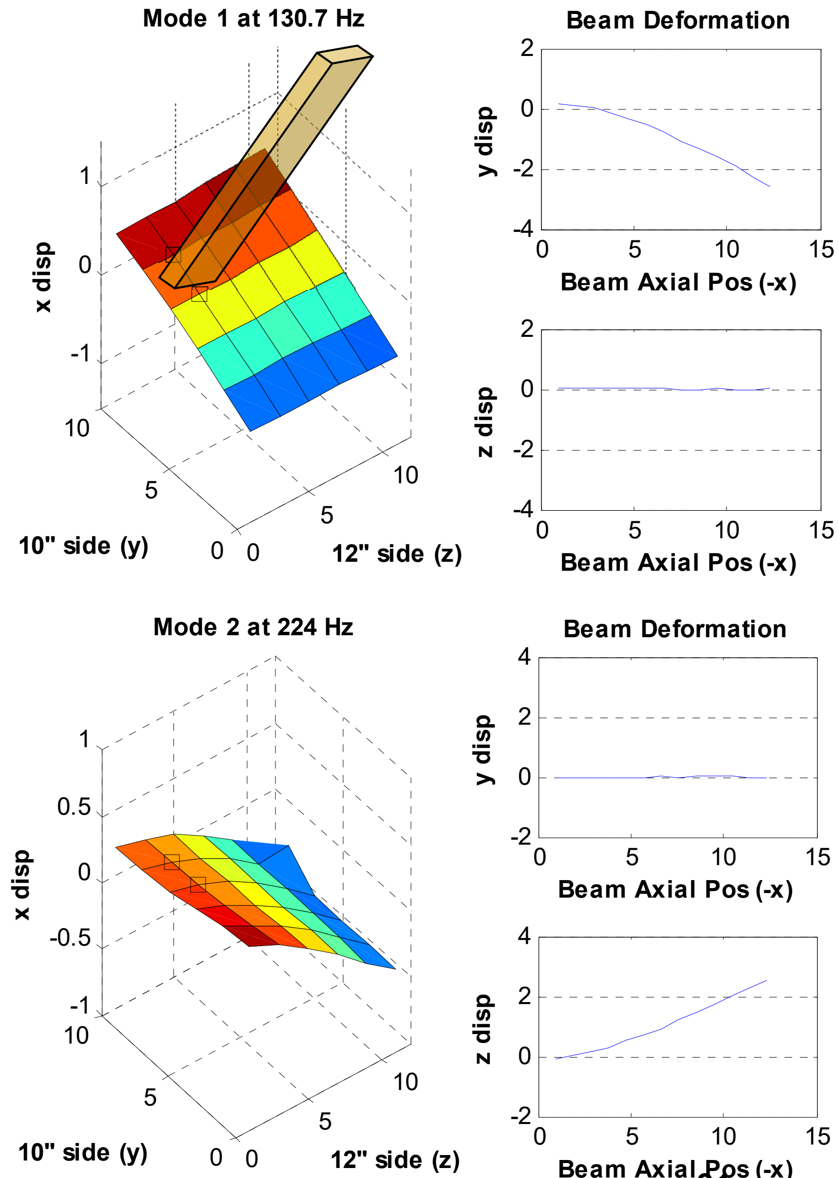
- Objective: Find the fixed-base modes of the beam.
- Experimental Design:
  - 3rd beam nat. freq. (2nd y bending) = 1st plate nat. freq.
  - 6th beam natural freq. (2nd z bending) = 3rd plate nat. freq.
  - The plate (fixture) is not much stiffer than the system of interest (beam); this would not be an optimal fixture design.
- Assures that the modes of the two systems are tightly coupled creating a challenging test for the methods.

Measurement Points

$$N_o = 48 + 26 = 74$$

Plate+Beam Modes

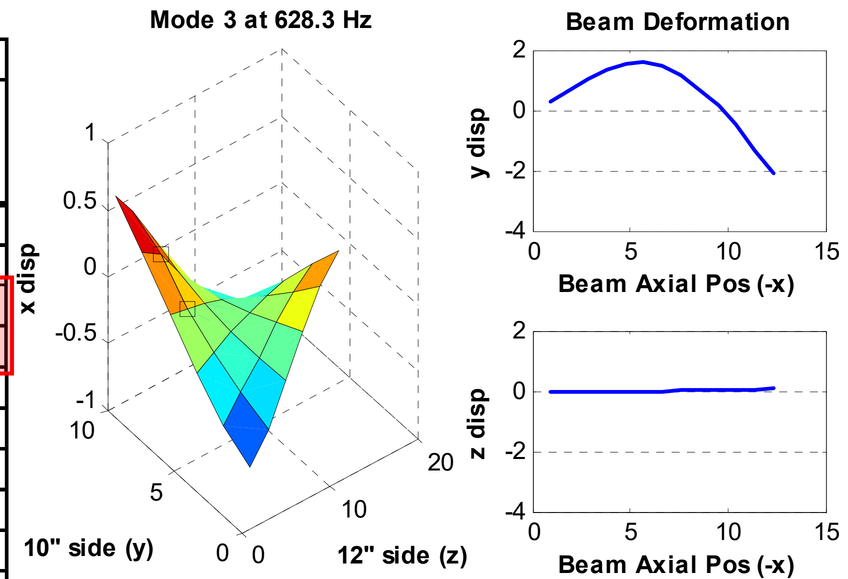
Nat. Freqs (Hz) of Elastic Modes				
Mode	**Plate Alone (Fixture)**	**Mode**	**Plate + Beam**	**Plate+Beam Modes:**
1	670.5	**1**	130.7	Beam 1y
2	893.7	**2**	224.0	Beam 1z
3	1344.0	**3**	628.3	Beam 2y + Plate
4	1620.3	**4**	693.8	Beam 2y + Plate
5	1850.4	**5**	902.4	Plate
6	2538.0	**6**	1254.4	Beam 2z + Plate
7	3107.8	**7**	1350.2	Beam 2z + Plate
8	3143.6	**8**	1657.8	Plate
9	3591.1	**9**	1770.7	Beam 3y + Plate
10	4082.0	**10**	1797.8	Beam 3y + Plate
11	4814.5	**11**	1906.1	Beam 3y + Plate
12	4837.7	**12**	2311.9	Plate
13	5220.6	**13**	2995.7	Plate
14	5561.4	**14**	3107.7	Plate
15	6765.1	**15**	3233.0	Plate
16	7093.4	**16**	3424.5	Beam 3z + Plate
17	7147.8	**17**	3522.8	Beam 4y + Plate
18	7179.0	**18**	3845.5	Plate
19	7808.4			
20	8288.7			
21	8414.8			

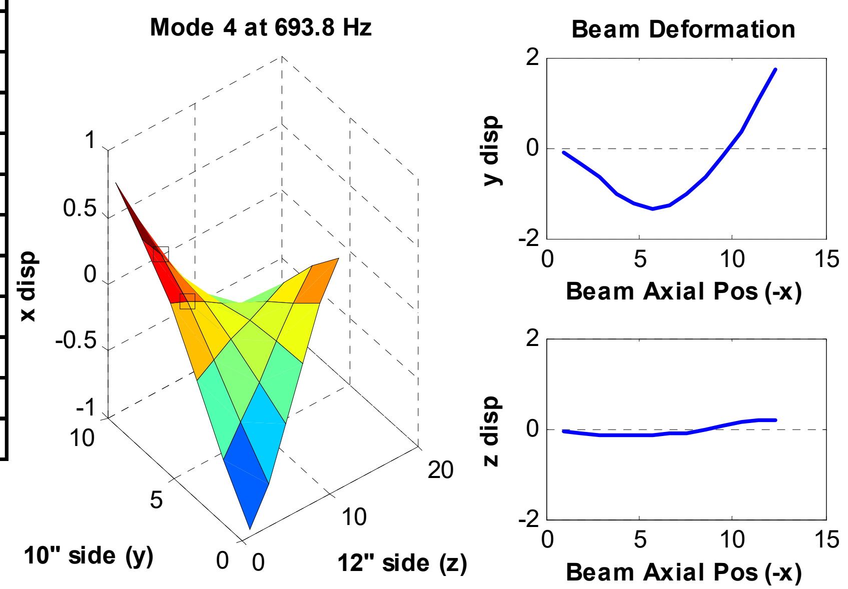


Plate+Beam Modes (2)

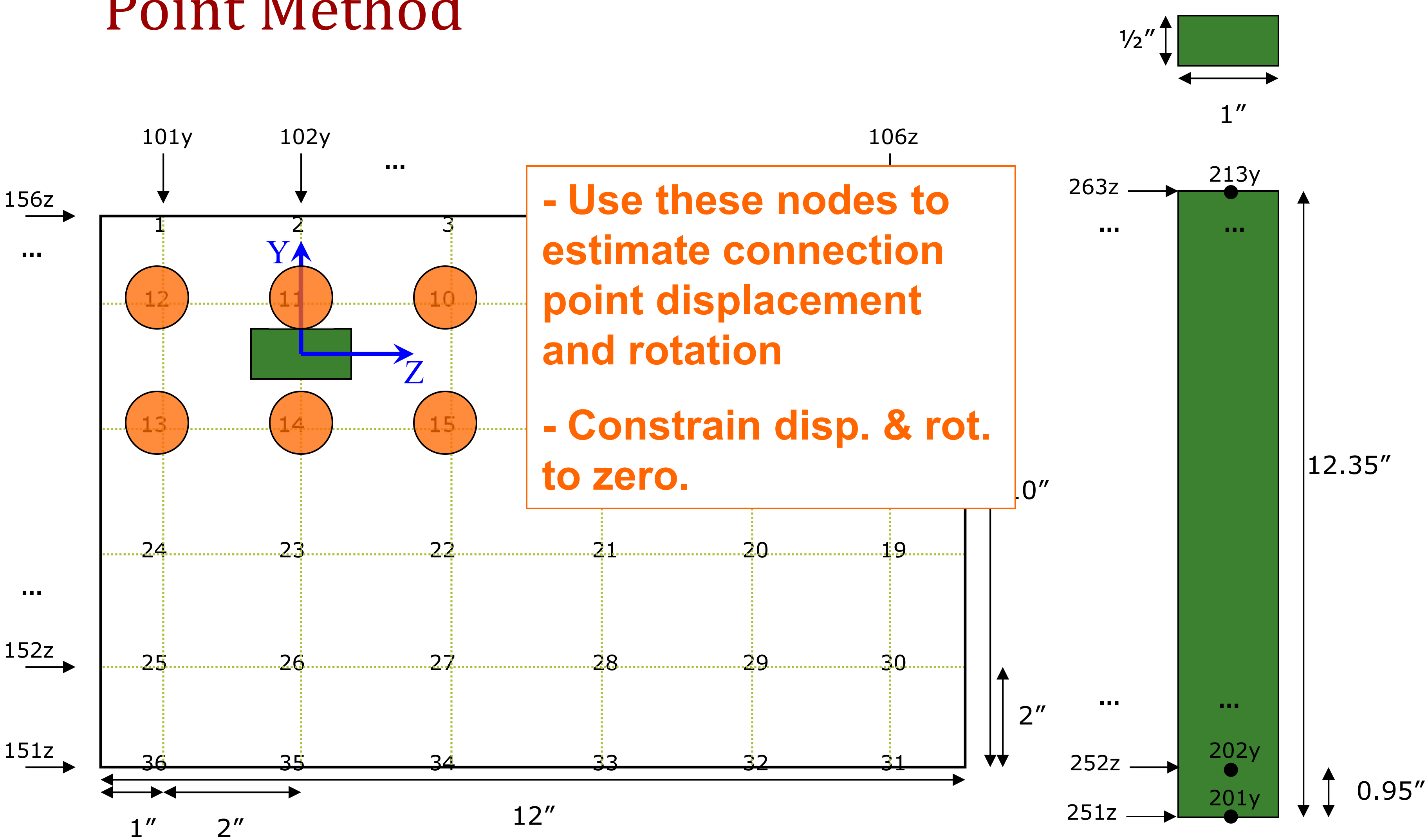
Nat. Freqs (Hz) of Elastic Modes				
Mode	**Plate Alone (Fixture)**	**Mode**	**Plate + Beam**	**Plate+Beam Modes:**
1	670.5	1	130.7	Beam 1y
2	893.7	2	224.0	Beam 1z
3	1344.0	3	628.3	Beam 2y + Plate
4	1620.3	4	693.8	Beam 2y + Plate
5	1850.4	5	902.4	Plate
6	2538.0	6	1254.4	Beam 2z + Plate
7	3107.8	7	1350.2	Beam 2z + Plate
8	3143.6	8	1657.8	Plate
9	3591.1	9	1770.7	Beam 3y + Plate
10	4082.0	10	1797.8	Beam 3y + Plate
11	4814.5	11	1906.1	Beam 3y + Plate
12	4837.7	12	2311.9	Plate
13	5220.6	13	2995.7	Plate
14	5561.4	14	3107.7	Plate
15	6765.1	15	3233.0	Plate
16	7093.4	16	3424.5	Beam 3z + Plate
17	7147.8	17	3522.8	Beam 4y + Plate
18	7179.0	18	3845.5	Plate
19	7808.4			
20	8288.7			
21	8414.8			

Modes 1-18 used subsequently, plus 6 rigid body modes from an FEA model.





Substructuring with Conventional Connection Point Method



- **Use these nodes to estimate connection point displacement and rotation**
- **Constrain disp. & rot. to zero.**

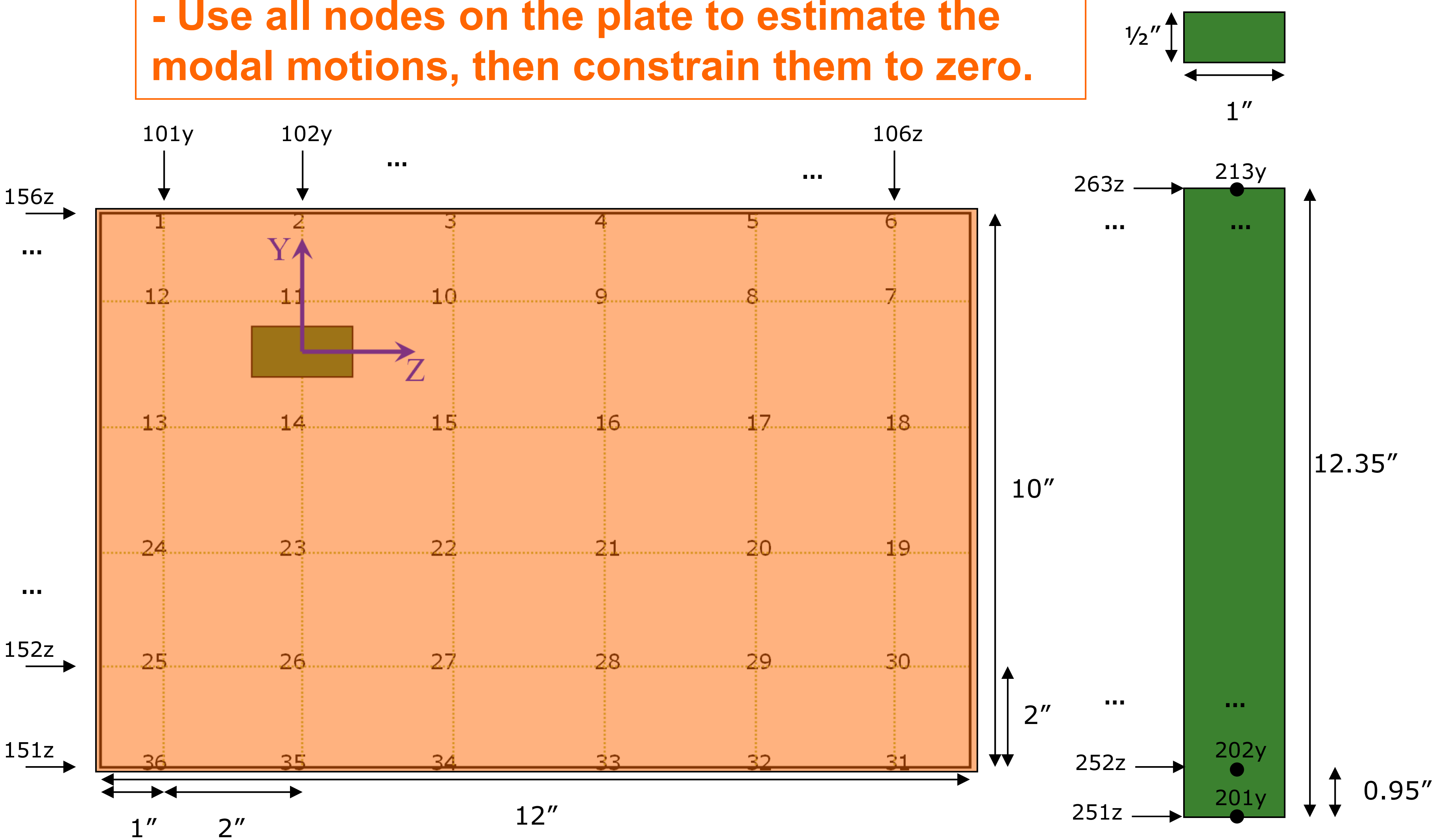
Conventional (CPT) CMS Results

Experimental Plate + Beam Frequency (Hz)	CMS Prediction using CPT Constraints	Analytical Fixed-Base Beam Freq. (Hz)	Percent Error in CMS Prediction
130.66	92.4	107.35	-13.9%
224.03	169.6	214.70	-21.0%
628.3	209.4	spurious	-
693.8	347.7	spurious	-
902.4	651.3	672.8	-3.2%
1254.4	665.2	672.8	-1.1%
1350.2	1234.8	1345.5	-8.2%
1657.8	1367.5	1345.5	1.6%
1770.7	1666.7	1883.4	-11.5%
1797.8	1778.8	1883.4	-5.6%
1906.1	1830.7	1883.4	-2.8%
2311.9	2111.5	spurious	-
2995.7	2870.4	spurious	-

- 1st two modes of the fixed-base beam are reasonably well estimated.
- Multiple estimates are obtained for the higher modes.
- Spurious modes remain in the frequency band of interest. These should have been removed by the CMS procedure (due to the constraints).

Substructuring with Modal Constraints

- Use all nodes on the plate to estimate the modal motions, then constrain them to zero.



Modal Constraints (12p/24pb)

Experimental Plate + Beam Frequency (Hz)	CMS Prediction with 12 MCFS Constraints	Analytical Fixed-Base Beam Freq. (Hz)	Percent Error in CMS Prediction
130.66	104.1	107.35	-3.0%
224.03	195.2	214.70	-9.1%
628.3	651.4	672.8	-3.2%
693.8	1230.5	1345.5	-8.6%
902.4	1777.1	1883.4	-5.6%
1254.4	1823.0	3691.4	-50.6%
1350.2	2369.4	3766.8	-37.1%
1657.8	2923.8	6102.2	-52.1%
1770.7	3045.3	7382.9	-58.8%
1797.8	3352.8	9115.6	-63.2%
1906.1	[illegible]	[illegible]	-71.3%
2311.9	3620.2	12731.7	-71.6%
2995.7	-	-	-

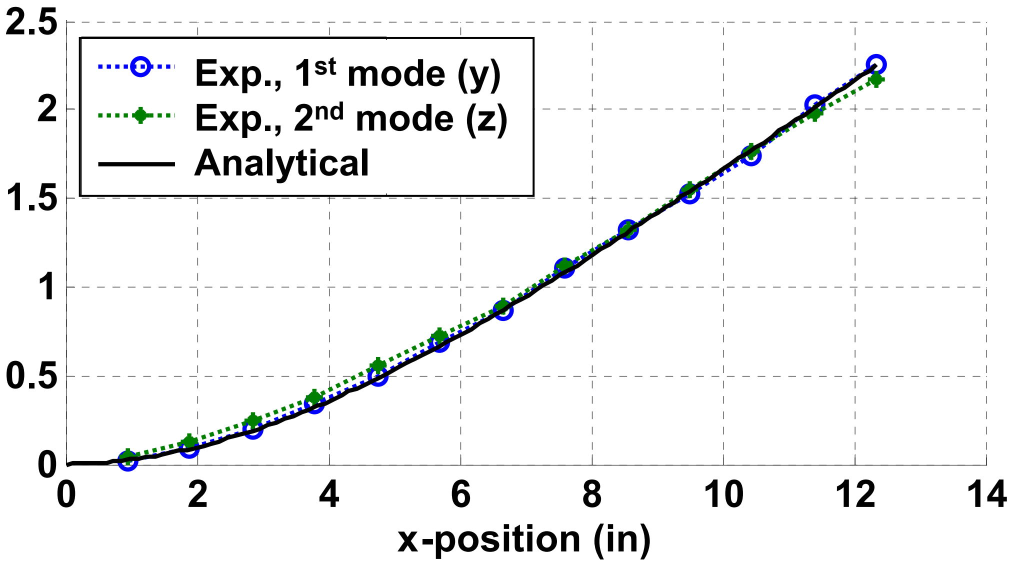
Bandwidth of Substructures:

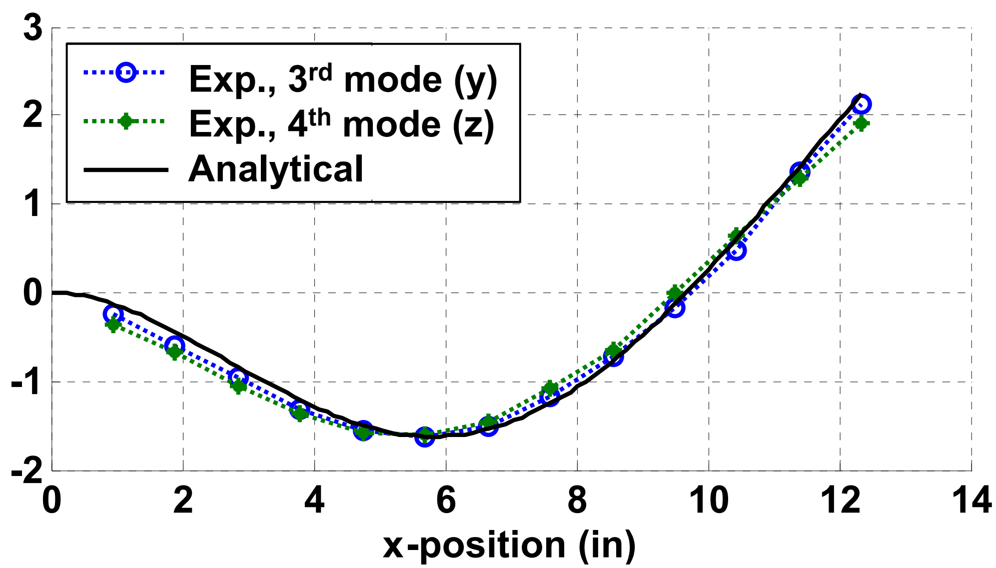
24 Modes (Plate+Beam)=4 kHz

12 Modes Plate alone=3 kHz

- CMS results found by coupling **12 experimentally measured plate modes** to **24 experimental plate + beam modes**.
- Six fixed-base modes of the beam are reasonably estimated.
- Modes with y-direction bending estimated more accurately (less stiff).

Substructuring Results (12p/24pb)

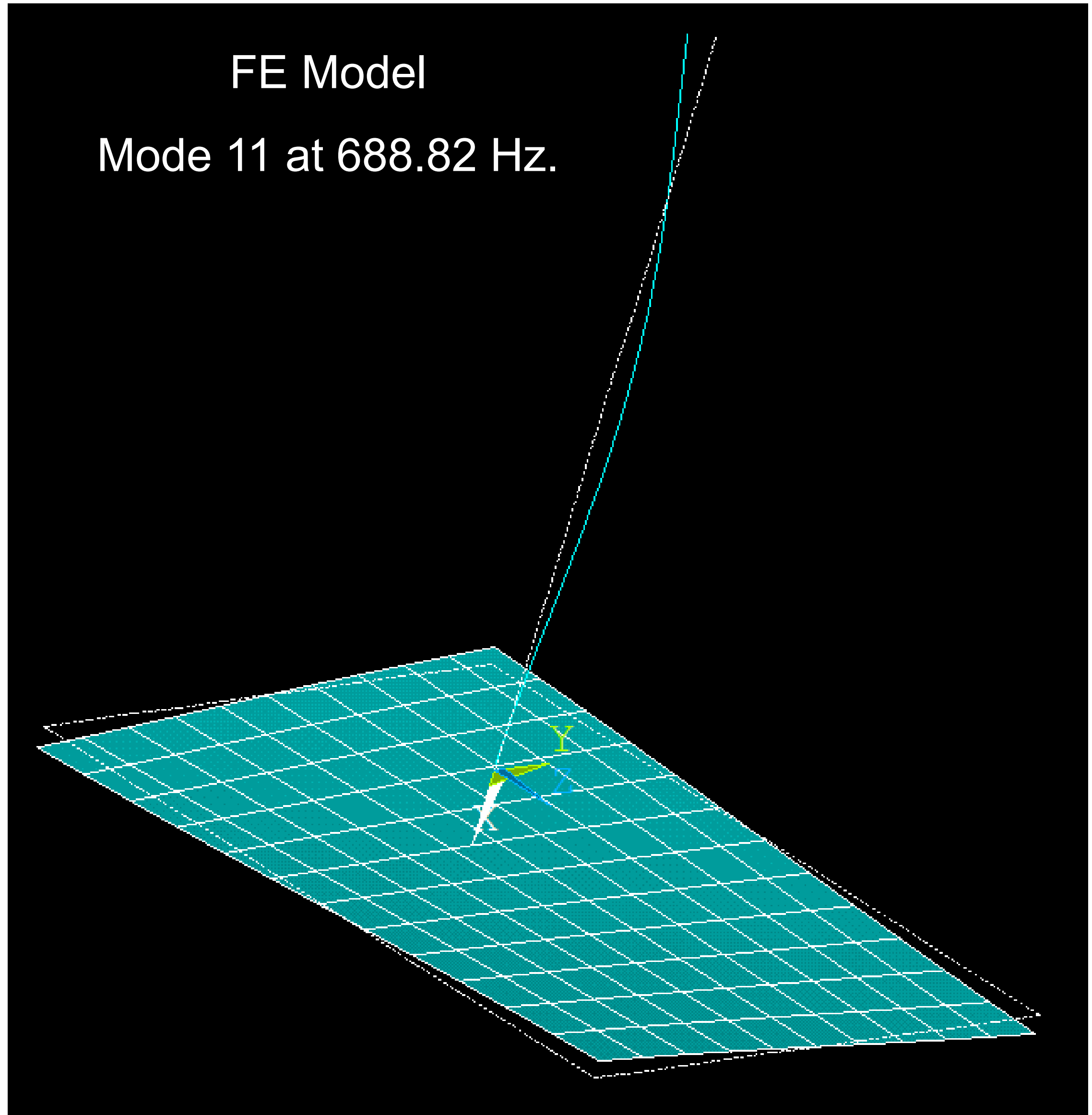




- Mode Shapes of 1st and 2nd y- and z- bending modes
- CMS Prediction using measured modes for plate and plate+beam.
- Both modes predicted very accurately.
  - Small difference in shapes perhaps due to slope near the root of the beam.
  - Why are frequencies not more accurate?

Analytical Results

- A simple finite element model was created and used to compute the CMS results that would be obtained with perfect measurements.
- Plate meshed with 1.0 inch square 8-node shell elements.
- Beam meshed with 2-node beam elements.
- Mode shapes of first 30 modes of plate and plate+beam found and exported to Matlab.



Analytical Substr. Results (12p/24pb)

FEA Plate + Beam Frequency (Hz)	CMS Prediction with 12 MCFS Constraints	Analytical Fixed-Base Beam Freq. (Hz)	Percent Error in CMS Prediction
126.34	103.25	107.35	-3.8%
207.2	183.8	214.7	-14.4%
630.96	647.53	672.77	-3.8%
688.82	1180.5	1345.5	-12.3%
899.33	1810.4	1883.4	-3.9%
1207.9	2431	3691.4	-34.1%
1349.2	2930.6	3766.8	-22.2%
1641.3	3072.3	6102.2	-49.7%
1786.1	3345.6	7382.9	-54.7%
1895.3	3389.2	9115.6	-62.8%
2316.9	3536.2	12204	-71.0%
2996.3	4173.8	12732	-67.2%

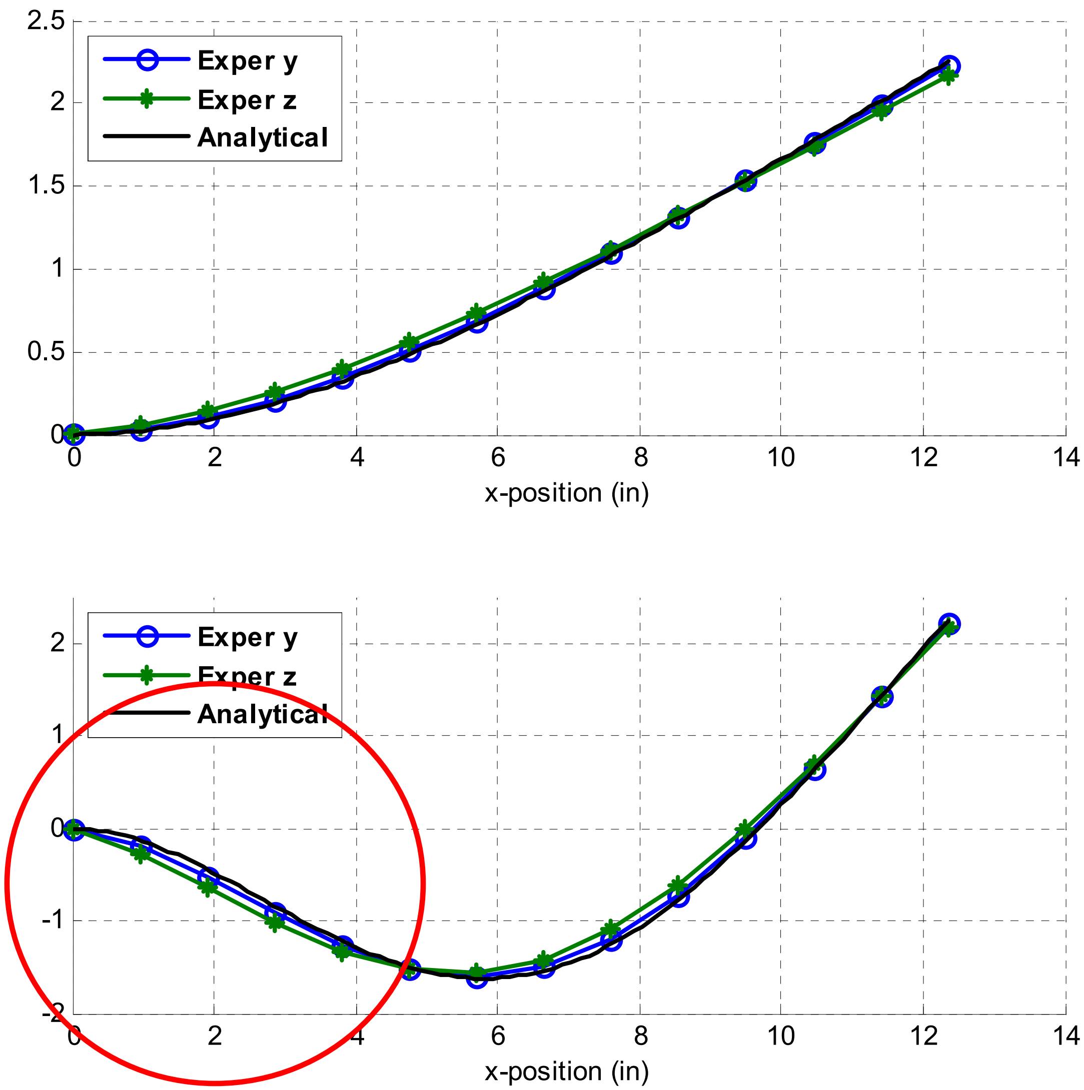
- Results using CMS with FE modes show a similar level of error as obtained experimentally!

Effect of Number of Modal Constraints

FEA Plate + Beam Frequency (Hz)	CMS Prediction of Fixed-Base Frequencies (Hz) vs. # of Modal Constraints							Analytical Fixed Base
	6	8	10	12	14	16	18	
126.34	102.07	102.65	103.04	103.25	103.27	103.28	107.52	107.35
207.2	175.91	181.84	183.15	183.8	184.45	184.52	202.39	214.7
630.96	623.49	643.32	646.2	647.53	647.63	647.68	679.08	672.77
688.82	683.91	1157.6	1176.6	1180.5	1183.5	1183.8	1267.4	1345.5
899.33	892.38	1244.7	1674.6	1810.4	1810.8	1811	1929.9	1883.4
1207.9	1181.3	1634.8	1825.4	2431	2446.4	3075.1	3421	3691.4
1349.2	1253.2	1689.1	2198.2	2930.6	3326.9	3332.9		3766.8
1641.3	1636.3	1826.3	2802.7	3072.3	3383.5	3535.4		6102.2
1786.1	1690.1	2235.3	2933.3	3345.6	3535.2			7382.9
1895.3	1830.4	2822.3	3072.5	3389.2	4116.6			9115.6
2316.9	2262.5	2943.6	3347.6	3536.2				12204
2996.3	2838.4	3072.6	3425.1	4173.8				12732

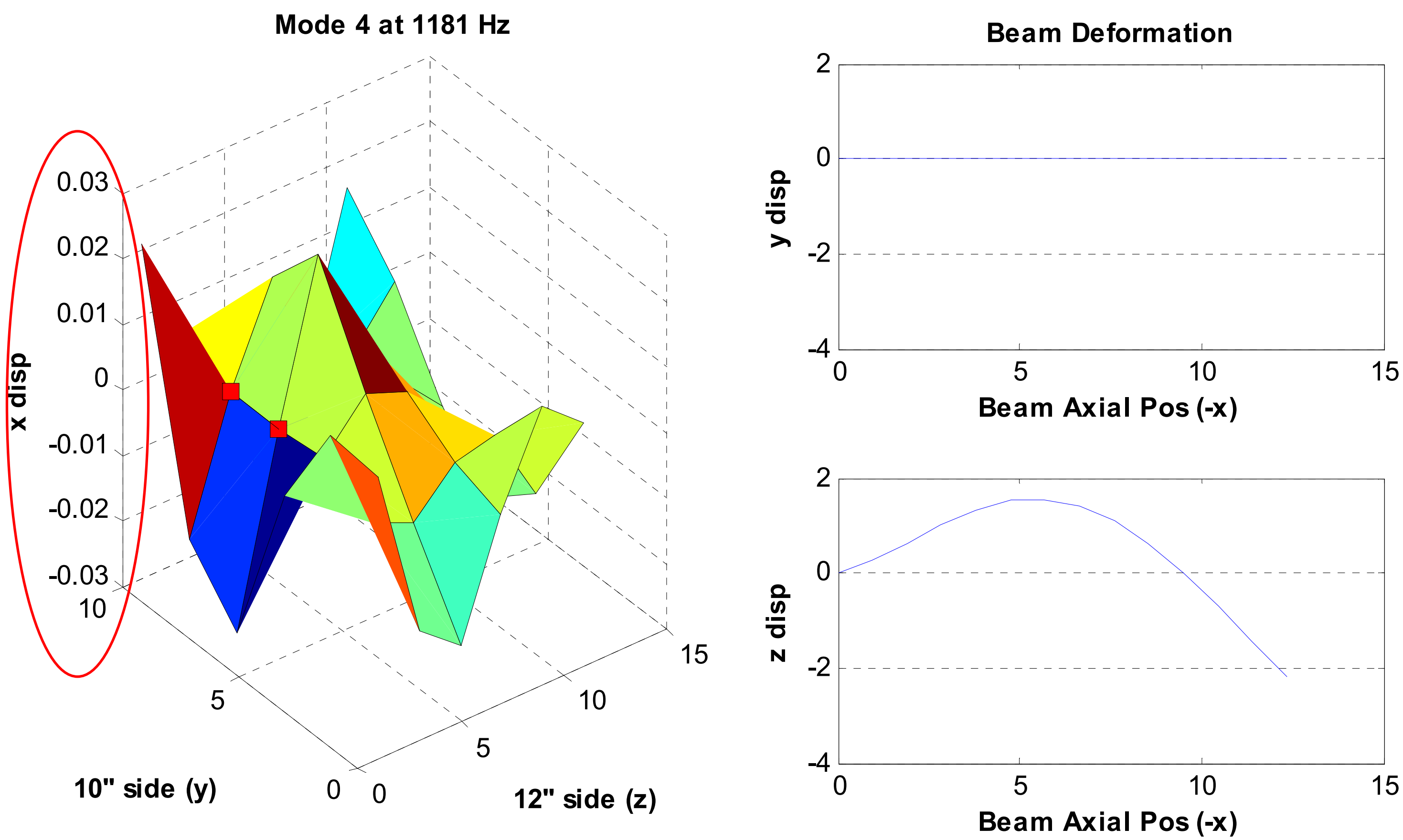
- Natural frequencies converge very slowly as the number of modal constraints increases.
  - Exception: with more than 16 constraints too few modes remain in the system to describe its motion, so the results change dramatically.

Analytical Substr. Results (12p/24pb)



- As with experimental results, the mode shapes agree quite well with the analytical shapes for a cantilever beam.
- But, is there some residual rotation at the base of the beam?

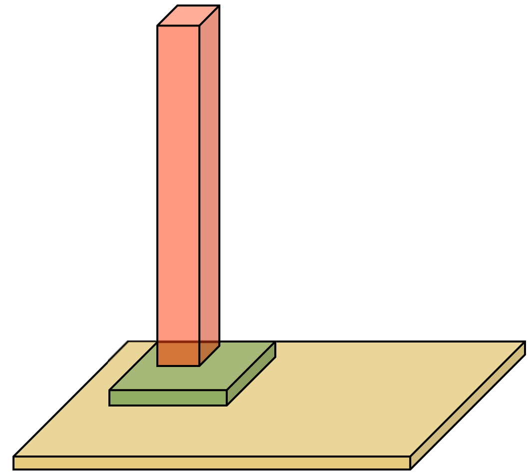
Mode Shapes of Plate+Beam **after Coupling**



- Some residual rotation is visible at the base of the beam in the bending direction in each of these modes.
  - But, the observed residual rotation is not large enough to cause a significant deflection of the tip of the beam.

CMS with Modified Fixture

FEA Plate + Beam Frequency (Hz)	CMS Prediction with 12 MCFS Constraints	Analytical Fixed-Base Beam Freq. (Hz)	Percent Error in CMS Prediction
122.00	107.0	107.35	-0.3%
251.39	211.9	214.70	-1.3%
667.0	669.2	672.8	-0.5%
778.1	1317.3	1345.5	-2.1%
980.9	1867.4	1883.4	-0.8%
1299.1	3211.4	3691.4	-13.0%
1524.4	3539.4	3766.8	-6.0%
1819.7	3638.7	6102.2	-40.4%
1896.4	3652.5	7382.9	-50.5%
2173.6	3768.2	9115.6	-58.7%
2898.0	3904.8	12204.3	-68.0%



- Analysis repeated using FEA model with the section near the base of the beam <u>three times as thick</u>
- Errors reduce significantly!
- **Conclusion: Frequency errors caused by local deformation of the plate that is not captured by the free modes of the plate!**

Alternative: SVD Modes

Doesn't require an estimate for the free-modes of the plate!!!

- In some cases one may not have an adequate fixture model. This shows what can be done with the measured shapes only.
- This slide shows the result of using the dominant shapes observed on the fixture (for a test of fixture+structure) to form constraints.
  - The rigid body modes were used specifically to assure that all of the rigid body motion disappeared.

Experimental Plate + Beam Frequency (Hz)	CMS Prediction with 6-RB & 9 SVD Constraints	Analytical Fixed-Base Beam Freq. (Hz)	Percent Error in CMS Prediction
126.34	103.3	107.35	-3.8%
207.20	182.2	214.70	-15.1%
631.0	647.6	672.8	-3.7%
688.8	1169.0	1345.5	-13.1%
899.3	1806.8	1883.4	-4.1%
1207.9	1835.3	1883.4	-2.6%
1349.2	2793.2	spurious	-
1641.3	3339.5	3691.4	-9.5%
1786.1	3535.6	3766.8	-6.1%
1895.3		-	-
2316.9		-	-
2996.3		-	-
3119.9	-	-	-

24 P+B – 15 = 9

Modes of Fixture+Structure

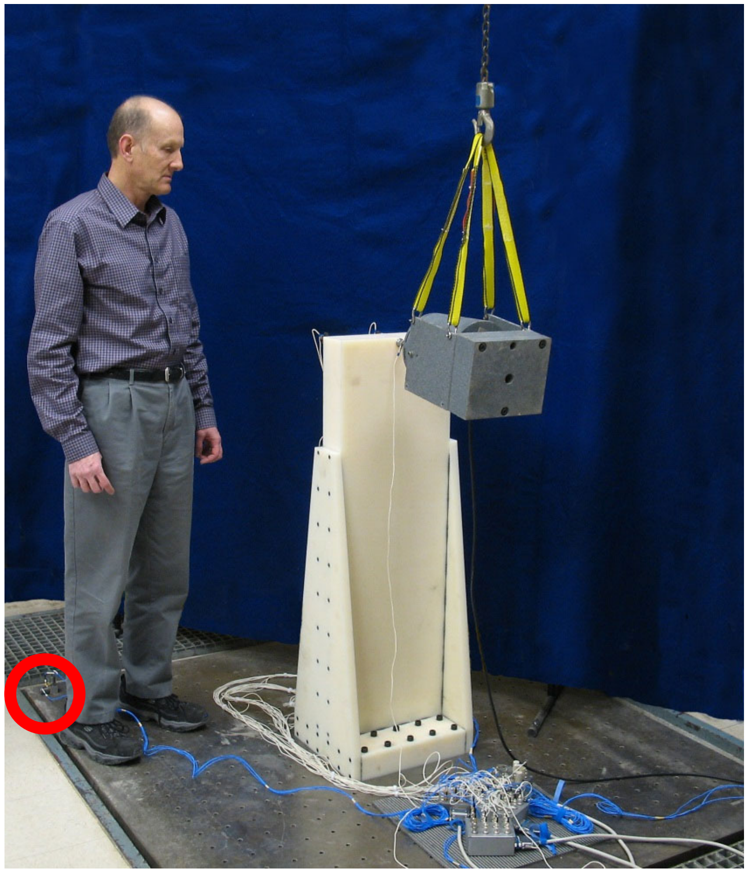
- Constraints:
  $\Phi(:,7:24) = U*S*V^T$
  $[\Phi(:,1:6), U(:,1:9)]^+\{y_f\} = 0$
- Results are comparable to those obtained with free-modes of the plate, but again the frequency errors are a concern.
- Again, convergence is very slow as the number of constraints increases.

Application to Beam (Sandia)

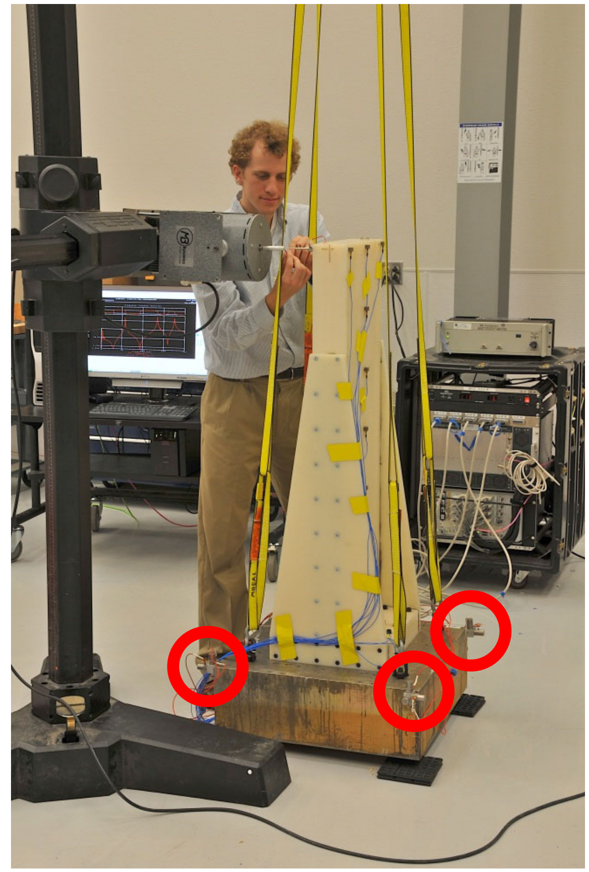
Another Example: Slip Table Modal Test

- We desired to find the fixed base modes of a 72kg nylon beam mounted on a magnesium slip table with oil film riding on a granite block
- We performed a modal test with a 1125 N shaker and force gage and instrumented the slip table with 7 triaxial accelerometers which would later be constrained to achieve fixed base results
- We also needed a “truth test” to compare the fixed base modes against

Truth Test Hardware – Big mass for modes up to 250 Hz and smaller seismic mass for modes from 250-1380 Hz



Large Seismic Mass Test



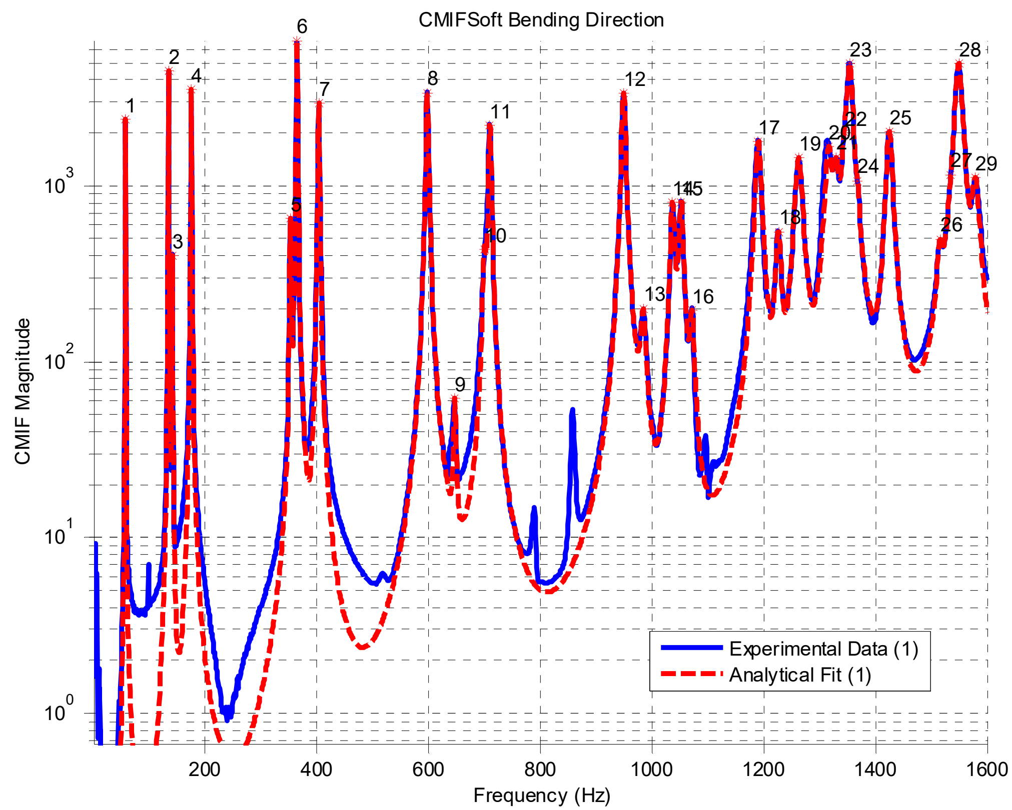
Small Seismic Mass Test

Truth Test Results

- In both tests on seismic masses we tested with three orthogonal shakers with burst random to improve damping estimates
- Φ_c was calculated from just the six rigid body modes on the large seismic mass with four triaxes on each corner of the mass
- Φ_c was calculated from the six rigid body modes and first elastic bending mode (1380) Hz on the small seismic mass

Modal Test Results – Small Seismic Mass

- CMIFs (blue) for soft bending direction shown below with analytical CMIFs (red) generated from extracted modal parameters



Truth Test Results (Results in Green)

Description	Unconstrained Small Seismic Mass		Constrained Small Seismic Mass		Constrained Large Seismic Mass	
	Frequency	Damping	Frequency	Damping	Frequency	Damping
1st bend soft direction	57.85Hz	0.18%	38.99Hz	0.16%	38.54Hz	0.22%
1st bend stiff direction	135.18	0.21	89.05	0.18	91.13	0.19
1st torsion	142	0.25	134.31	0.21	134.0	0.20
2nd bend soft direction	175.75	0.24	163.61	0.22	164.0	0.24
2nd bend stiff direction	353.18	0.33	338.16	0.32	342.3	0.25
2nd torsion	364.91	0.26	364.24	0.26		
1st Axial	409.31	0.26	392.4	0.25		
3rd bend soft direction	403.90	0.32	396.4	0.31	394.8	0.32
3rd torsion	598.1	0.42	597.6	0.42		
3rd bend stiff	647.3	0.61	643.8	0.61		
4th torsion	700.5	0.47	700.4	0.47		
4th bend soft	710.4	0.37	706.4	0.37		
4th bend stiff	791.6	0.51	790.4	0.51		
5th bend soft	859.2	0.29	859.2	0.29		
5th torsion	949.5	0.41	949.2	0.41		
6th torsion	984.5	0.51	984.3	0.51		
6th bend soft	1037	0.43	1034.8	0.43		
7th bend soft	1052.5	0.43	1048.2	0.43		
2nd axial	1073	0.40	1062.3	0.39		
5th bend stiff	1099.6	0.39	1097.2	0.39		
8th bend soft	1190.7	0.49	1190.7	0.49		
7th torsion	1262.7	0.60	1263.1	0.60		
9th bend soft	1316.6	0.54	1316.1	0.54		
8th torsion	1330	0.45	1330.7	0.45		
10th bend soft	1344.3	0.51	1344.3	0.51		
11th bend soft	1354	0.47	1352.3	0.47		

- Note that there is another example of fixed base substructuring using FRFs from IMAC 28 in 2010, paper 236 by Mayes, Bitsie & Bridgers.